Geurino Mazzola on Music Theory

(1) Tuesday, March 15: Salle "Simone Weil" at the ENS

Terminology: Moebius strip:
"Rubin Harmonique"
"Lien Harmonique"

pg. 42 of Schoenberg's Harmonielehre: "Hexachords"

The Moebius strip is of course a double cover over the circle, somehow he interprets the Major-Minor system as a double cover over the Diatonic System. If one draws the chart of connections of triads (such as, I recall, Schoenberg does have in his Harmony text) one gets a Moebius strip structure. In other words, "modulation" operates as a Moebius strip of association of chords. The "cadential formulæ" are simply mappings of the Moebius strip into itself.

$S^3 \rightarrow T^3$. Cadence and symmetry. Okay. It does make sense but I don't know how difficult it's going to be to communicate it.

Fill out the chart of connects developed from the group operations on this diagram:

\[
\begin{array}{ccc}
I & III & VI \\
IV & & V \\
\end{array}
\]
Mazzola's "quantum" (fundamental mass notion) A collection of notes possessing internal symmetries. These symmetries are intended to transmit an "invariant" or "quantum" from one key to the next in modulation. (He's referring to a generalization of the technique of enharmonic modulation)

There are modulations without a quantum

Mazzola’s Theorem: There are 114 different basic formulae or patterns or "types" of modulation. All modulations from a major key to a major key preserve a quantum element.

Question: Does this result apply to "extended" harmony such as one finds in Schubert and Wagner? In fact Mazzola has never applied this analysis to actual pieces, not even simple ones by Bach.

Now Mazzola discusses Fuxian Counterpoint, and something which he calls the "Anthropic Principle in Music". Wanders off into a digression about Penzias and Wilson, George Gamov, the Big Bang, Hawking and Fowler, Teilhard de Chardin, the noosphere, and the "first musicians' which he locates in 50,000 BC.

1923: First atonal pieces. The "Big Bang" of Atonality!

Pompously quotes Heidegger:

"Why is there something rather than nothing?"

(Mostly blah-blah, but fun anyway)

Cites what is clearly an important reference:

Klaus-Jurgen Sachs: "Der Contrapunktus in 14 und 15 Jahrhundert"

Here Mazzola's work does indeed become interesting. Based on his read of K-J Sachs, he demonstrates that the "physics of
"consonance/dissonance" is very different from the way in which the consonance/dissonance system evolved in the 14th and 15th centuries, which leads to the system promulgated by Fux.

Mazzola presented a curious result in modular arithmetic, which replicates the Fux system. In this we must take the "4th" as a dissonance, which it is in 3-part counterpoint, though not in 3-part.

Consider the equation \[ y \equiv 5x + 2 \pmod{12} \]

This has the strange property of transforming dissonances into consonances! Thus, using the customary system for intervallic representation, one has

- 1 = semitone; dissonance
- 2 = whole step; dissonance
- 3 = minor third; consonance
- 4 = major third; consonance
- 5 = fourth; dissonance
- 6 = tritone; dissonance
- 7 = fifth; consonance
- 8 = minor sixth; consonance
- 9 = major sixth; consonance
- 10 = minor seventh; dissonance
- 11 = major seventh; dissonance
- 12 = octave or unison; consonance

Clearly it is invertible, since \[ 5(5x + 2) + 2 \equiv x \pmod{12} \]

Mazzola relates this phenomenon to his Moebius strip. A trip around the strip takes us from consonance to dissonance. Another trip brings us back to the beginning.
II. Tuesday, March 22:

Standard reference: "Topos of Music"

Project: To use modern mathematics for the analysis of musical performance. Mazzola takes strong exception to the Schenkerian viewpoint whereby music exists in some Platonic realm of ideas apart from its performance. Performance here is defined as "Expressive interpretation based on an understanding of the text."

Mazzola then launched into a description of a musical score as a kind of manifold with overlapping neighborhoods, within each of which there is a particular interpretation supplied by the performer.

Another quote, Paul Valéry:

"C’est l’execution du poème qui et le poème."

Quote from Adorno:

"Performance is essential, not incidental, to music."

Historically all composers were musicians. The divergence between the two functions is a very recent phenomenon. Even as recently as Bach (organ improvisations) Chopin and Mozart (piano concertos), etc., what we have in printed form today is a frozen version of something originally elaborated in the concert situation. This is particularly true in the case of Chopin, from whom we have often numerous differing manuscripts for the same piece.

Mazzola's "manifold" concept is introduced to allow for the introduction of "infinitesimals" and "interpolations". Speaks of a quantum notion as well, a "minimal quantity."
States that the "best" interpretation is the "most precise" interpretation. To quote:

"Interpretation should be rendered explicit down to the notes and the intervening silences."

Forms of Expression:

1. Emotions (Cites Alf Gabrielson)
2. Gestures
3. Analysis (Schenker, Adorno and, of course, Mazzola)

Usual tripartition of Intellect, Heart and Body.

Sets up a 3D Cartesian representation in which notes can be represented a point in space:

Seems to be speaking of an "interpretation" as a kind of "diffeomorphism" of this space! Brings in the notion of a "discretized manifold". Here is an essential weakness of his methodology, because although music is written down in a discretized notation, what makes it "music" are the connections between notes, or, let us say, events. This connectivity requires a new kind of manifold.
He does speak of the "idea" of continuity as being projected by "discrete" means.

His manifold eventually finds its way into a Fast Fourier Transform, or wavelet analysis of the performance of a small part of a piece. He's constructed an instrument that displays this. The multicolored picture could conceivably allow a music teacher to work with a student in improving the shape of the profile of this analysis.