Mathematics and the Modeling of A Priori Assumptions

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Examined dispassionately, the world-wide "scientific enterprise" as practiced today, with its over-emphasis on technology and disdain of philosophy, can be understood as a crusade to compartmentalize the entire universe, to turn it into a bureaucracy.

To this end, the "pure", "disinterested", or "hard" sciences have been incorporated as wholly owned subsidiaries of the universities and technical colleges. Thus science, which strives to compartmentalize nature, becomes itself institutionalized.

This hierarchic template is further extended by the bureaucracies of science: the National Science Foundation, CNRS, New York Academy of Sciences, governmental agencies such as the Department of Energy, NASA, NIH, NSF, etc. , the AAAS, the professional societies ...

That is to say: ***the institutionalization of the bureaucratization of the institutionalization of the compartmentalization of the universe!!***

Given this dense overlap of controlling mechanisms, where can one

find any space left for Nature itself? Is this multi-dimensional tinker toy, this entire Jungle Jim of graphs, networks and frameworks, all that we can truly know, possess, call our own? Must we therefore conclude that, with Immanuel Kant, the *Ding An Sich* (The Truly Actual) is inherently unknowable? Are all the fish caught in our elaborate nets inevitably destined to escape? Let's take a closer look:

Mathematics, Physics and the Knowable Universe

The mathematical schemes which we employ in our attempts to understand the cosmos are simplifications drawn from the structure of the information contained in the sense data gleaned from observation and experiment. At the base of the chain, the observations themselves, afford only the tiniest glimpse into the makeup of the actual universe. From pitifully inadequate data we conjure up mathematical idealizations which *substitute for both the data and the universe they describe*, because they can be used to make predictions.

However there is a strong common bond between the eminently knowable "pure" mathematics and the unknowable but hypothesized

"actual" universe, in that these are both deemed *absolutely true* . However the observational sense data is intrinsically flawed and susceptible to gross errors. One need only recall the recent experiments at CERN which appear to show that neutrinos can travel faster than light (*NY Times September 23, 2011*).

The initial and terminal 'spaces' of the scientific process, *the space A of physical actuality* and *the space M of mathematics* both of which share the attributes of absolute truth, are united, as by a strong yet treacherous bridge, by *the space D of sense data* , inherently flawed, capable at most of falsification but never of confirmation.

These considerations are fairly obvious, yet they lead to non-trivial reflections when one considers the *choice of models* derivable from mathematics: Euclidean and other geometries, dimensionless points, hard impenetrable objects, fields, continua, harmonic oscillators, differential structures, analytic functions. These are the established bureaucracies, sometimes called categories, each of them the basis for entire academic fields and disciplines. Schematically:

Geometry

Dimensionless, point-like entities (Particles)
Linear entities (Strings)
Spatial entities (Classical Mechanics)
Objects in generalized N-spaces (Statistical Mechanics)

Algebra

Linear
Polynomial
Exponential
Symmetries (groups, etc.)

Analysis

Differentiable
Partial differential equations
Complex Analytic
Multi-variate

Arithmetic

Discrete
Continuous
Finite
Bounded
Unbounded
Infinite

In point of fact, the actual universe is NONE of these things; it is only in the mathematical interpretation of the limited "observational sense data" that one uncovers these structures. Yet the process does not

end there:

As a general rule, the mathematically derived entities ***will be further idealized*** **, either because the resultant equations can't be solved, or for the purposes of simplification, or to be able to introduce techniques of manipulation available from calculus ,the theory of differential equations, Fourier transforms, etc.**

Example: Statistical Mechanics

The huge number of particles of a gas that were hypothesized in the original papers in Statistical Mechanics were not observed at the time. Yet, even before atoms and molecules were confirmed by indirect, then direct observation, they'd ***already*** **been idealized to an infinity of particles, then to a manifold !!**

Why? Boltzmann wanted to justify mathematical equations involving both derivatives and integrals, such as the unmanageable H-Theorem equation. The transition from a finite number of particles to an unbounded number was done to give credence to the idea of a "phase space V volume W" and probality measures over V and W, while the

transition to an unbounded number and an infinite number (indeed uncountably infinite) was done to justify the use of derivatives and integrals.

At the same time there was a parallel development in the study of Brownian motion. Although Brownian motion had been observed, (and given an atomic explanation!) by Lucretius in 60 BCE, the mathematical description of it was supplied by Einstein in 1905. The direct observation of atoms came much later, with the invention of the electron microscope by Leo Szilard in the 1930's.

One sees at work a natural progression of:

(1) Mathematical structures
(2) Further idealizations of these structures
(3) Physical models drawn from the mathematics
(4) (Hopefully) Eventual direct observations of the entities from which these structures are derived

Each episode of the historical scenario has been accompanied with a corresponding collection of mathematical representations:

Sadi Carnot: **caloric theory. "Caloric" is a substance, thus a continuum.**

Boltzmann/Maxwell: **huge numbers of particles ,later idealized to an reified to an infinite number , ultimately to a 'manifold'**

Lucretius, Jan Ingenhouz, Robert Brown: **observation of Brownian motion**

Bachelier, Einstein, Smoluchowski : **A mathematical theory which proposes that the fluctuations of Brownian Motion are evidence for the existence of molecules and atoms**

Perrin (1909): **Correlation of the Einstein equation with direct observation of Brownian movements**

Szilard (1931) **:** *The electron microscope*

In these examples drawn from statistical physics, also typical of much of science, one attests to a proliferation of models combining the continuous and the discrete, multitudes of dimensionless points, fictive 'volumes' and 'densities' , a ratatouille of Newton, Riemann, Stieljes and Lebesgue integrals implicit in the Boltzmann equation of the H-function …. The list is long , and not without oxymoronic self-contradictions. "Nature" could care less: it is, as stated above, absolutely true though

essentially unknowable.

A Priori Assumptions

Work in the sciences would be completely unmanageable, were there not another component in the development of scientific understanding, one that Kant understood very well : ***the demand for intelligibility*****. Kant calls it the** ***synthetic a priori*** **and he places it squarely in the intersection between cosmos and mind. Succinctly, the observable world would not be** ***intelligible*** **if the mind did not insist on the existence causation, extension, continuity, homogeneity, transitivity of time and space…**

Kant may have been wrong about the a priori character of Euclidean geometry, but in our modern perspective one can perhaps rescue his schema by replacing global geometry by the differentiable, locally connected manifold. One can continue to debate endlessly the status of those principles which can be accepted as being synthetic apriori, or even whether such entities exist or are meaningful. In fact, such high-

level debates are a far cry from the very crude models used in the work of experimentalists, and even of theorists.

The basic argument of this article is that when mathematics is put to work in theoretical physics, ***the objects it is designed to model are a priori assumptions*** **, those mental images which make the subject of research intelligible. It** ***does not*** **model the data directly, but indirectly through providing an expressive representation of the ideas behind the interpretation of the data.**

Let me illustrate this hypothesis by applying it to various conceptions of causation.

Causation

Both Kant and Hume were in agreement that the "detection" of cause and effect cannot be made by any sense organ or measuring instrument. Yet science would be lost without causation. There are, of course, scientific philosophies that speak of universal laws as a collection of tendencies derived from essentially random behavior. To do so is to put Descartes before the horse.

Statistical mechanics starts from the assumption that the universal order is given a total description by classical mechanics. The theory was developed for the purpose of grounding Thermodynamics in Classical Mechanics. Probability is not the defining causal mechanism, but the expression of our own finite limitations.

Likewise in quantum mechanics, "uncertainty" does not apply to individual observables, but to complementary pairs of observables, or to a quantity that is not an observable of the natural universe: action.

Generally speaking, the concept of probability as employed in physics differs from the way it is used in mathematics. In a mathematical thought experiment, it is possible that the "heads" on a perfect coin never turns up at any toss . To a physicist, the statement that a certain phenomenon has a 50% probability means that over infinite time, the phenomenon *will* occur 50% of the time! How else explain the *determined correlations* of the non-locality experiments that verify Bell's Theorems?

Let us now examine 3 basic causal paradigms:

(A) Standard Causation for systems in isolation
(B) Universal Connectedness
(C) Renormalization of Infinities

(A) *Modeling the standard paradigm of physical causation*

The standard paradigm of classical causation is thoroughly discussed in my article "On the Algebraic Representation of Causation" available on the Ferment Magazine website at:

http://www.fermentmagazine.org/scipapers.html

The "Hamilton-Lagrange" paradigm as it is called in this paper, applies only to systems in isolation. We first state the paradigm, then we will examine what is meant by a "system in isolation".

The behavior of a system in isolation, for the amount of time that it remains isolated from the rest of the universe, is completely determined in both the forward and backward direction of time, by information that may be acquired in an infinitesimal neighborhood of any instant during the period in which it is isolated.

Conditions for a system in isolation

The requirements for a 'system in isolation' are such that this definition

appears to be (but is not quite) circular:

1. **The "system in isolation" S abides in a plenum P within in some idealized block in space, something like an "ideal laboratory".**
2. **The background state is unambiguous: normally what this means is that there is a strict separation between matter and space-time. (*This is old-fashioned, but the classical system in isolation is old-fashioned.*)**
3. **There are no influences coming from systems outside P. The only structures within this plenum are those of the universal conservation laws, matter, momentum, energy, light speed, etc.**
4. **There are no influences coming from the geometry of space-time, other than those which are uniformly present everywhere.**
5. **These conditions can be subsumed into a single condition: the behavior, throughout time past, present and future, of any system S contained within the plenum P , will not depend on any structure, system, state or matter outside of P.**

The "standard paradigm of causation" then states that there is

enough information present about S within P, in any infinitesimal neighborhood around any instant t, to be able to describe the behavior of S for all time, backwards and forwards, as long as it remains within P.

There is no guarantee that such an a priori assumption will automatically suggest a mathematics by which it can be represented. It is therefore all the more remarkable that there does exist a branch of mathematics, namely *the real domain/range of analytic functions of a complex variable*, **that fulfills this role!**

The models propounded by physicists, whenever possible, try to be based on analytic functions, ideally without poles. If there are poles, they function as singularities which can be derived from initial data. The best example is that of the singularity of the potential of a gravitational point source.

Analytic functions can be expressed as an infinite power series of of one or sever al complex variables. When combined in finite sets, they form analytic varieties which can model collisions and jump discontinuities without disturbing an essential smooth description of the

consmos. Thus the behavior of these *function varieties* on P, at any neighborhood of an instant of time, completely determine the behavior of S throughout all past and present time.

There are other possibilities for modeling standard causation by function algebras, which are discussed in "On The Algebraic Representation of Causation". They tend to be either far-fetched or pathological, though there may be roles for them, which I discuss. Yet no other class of functions does this as well as the analytic functions. What is important to note is that these functions (which after all, derive their properties from their behavior in the complex plane, not in the world of real quantities) *are far from being present in the sense data* . One looks for models based on analytic functions *because of a principle* , an *a priori assumption* necessary for intelligibility, namely that the universe, in small, connectible boxes (that is to say, in a manifold structure) is causal.

This is even true for quantum theory. The appropriate mathematics for quantum theory is that of a Hilbert Space over phase

(location/momentum) space. Each point, relative to a basis of orthogonal analytic functions, is a solution to the Schrödinger Equation. The square of the modulus of these functions is a probability density, propagated through time in a completely deterministic fashion. By the (somewhat hideous) Gestalt of the "collapse of the wave function", this probability magically jumps to 1, when the observation of the corresponding observable is confirmed!

Thus quantum theory has not abandoned causation, but replaced it with the "probability of observation". I dub this phenomenon "pseudo-causation". One is supposed to believe that, through fishing, guessing, consulting horoscopes, etc., one estimates fixes the location of a particle with probability k% . If one is lucky enough to find the particle there, k% jumps to 100% and the wave function collapses instantly throughout the entire universe. (Instants themselves being dimensionless points, thus not present in the “real” universe.)

Whether this is what really happens is inherent neither in the data, nor in the enveloping mathematics. These two perspectives must be further

combined with the notion that the universe cannot be made intelligible without positing some kind of causation.

(B) *The Kant-Leibniz paradigm of universal connectedness*

This view of causation is the dual to the standard paradigm. It assumes that in our real universe, there can be no causally disconnected systems, thus , neither in fact nor in principal, any systems in isolation. Given two systems physical systems S_1 and S_2 with compact support in space-time, that is, of finite extension, let us imagine that a hand can come out of the sky and perturb S_2. By the paradigm of universal connectedness a real perturbation of some kind in S_1 must inevitably occur.

A restricted version of this global perspective is present in General Relativity, provided that one allows for a time delay for the propagation of the perturbation. However, owing to the universality of gravitation (*one cannot build a wall of any kind to separate the gravitational attraction between two massive bodies* **) there can be no hand coming from the sky.**

Question: Are there forms of mathematics that naturally incorporate universal connectedness, in the same way that analytic functions represent standard causation?

Any kind of global structure, such as for example, the universal curvature that determines the gravitational constant, or the cosmic inflationary field, or dark energy, implies a form of universal connectedness. None of them have that totality envisaged by Leibniz, for whom distance is merely a secondary attribute of matter, or for Mach, to whom inertial mass itself was a function of the universal distributrion of matter.

A few more things can be said about the relationship of General Relativity to universal connectedness. According to General Relativity, the local metric incorporates influences coming from everywhere. If, before the Big Bang, the entire universe was compressed to a dimensionless point, then this connectedness between, say a distant quasar and an observer on earth, should still pertain. But if gravitation is all penetrating and can't be blocked, there can be no transcendent hands.

Therefore the connections dating back to the beginning of time are deterministic. Total connectedness then becomes something that can't be verified experimentally because there can be no outside perturbing influences.

This is why Hawking, Ellis and Penrose can speak confidentally of a "Large Scale Structure of Space Time", even though, in our earthbound state, we are handicapped by a pitifully inadequate window of vision on the entire universe.

(C) The Renormalization of Infinities

Under a "no infinities" assumption for the real, or knowable universe, it is common practice in physics to simply replace an infinity that pops up in mathematics by a finite quantity obtained from measurement.

It is intrinsic to any meaningful definition of physics, *as a science* **, that there can be no infinite quantities in nature. Physics is based on observation, and observations must ultimately be quantifiable, that is, measurable either in theory or practice. Even theoretical entities must be**

amenable to practical measurements if they are to be relevant to science. Virtually by definition, no infinite magnitude can be quantifiable, and certainly not measurable.

The difficulties come at the other end, that is to say, the existence of infinitesimals, of infinitely divisible entities, precise locations, or ratios in which the denominator is permitted to go to 0: continuous substances or infinitely divisible space, dimensionless points and instants, infinite densities or accelerations.

For example, can one speak of an "infinite acceleration" present at the instant of collision between two hard, perfectly elastic bodies? Leibniz thought that there could not, and imagined a steep yet continous bending of each of the pathways of the two particles during the collision event. Remarkably, collisions can also be given a smooth, analytic interpretation as the pair of points on the variety composed of the equations of the conservation of momentum and the conservation of energy.

There are many people who hold to the view that the point-center

of a Black Hole has an infinite density, But this conclusion is derived from an abstract ratio based on the Schwarzschild equation, it cannot be an observation. If it were possible to replace Nature by equations, one would be able to identify an inexhaustible treasure house of "real" infinities!

Classically, physicists and mathematicians would seem to believe, implicitly or explicitly, that infinitesimals and infinites could exist in real space and time, although not in matter, energy, momentum or radiation. Yet in our contemporary jumble of dependent and independent variables, time and space become dependent on matter and energy, and if the latter are finite then one can presume that the former must also be finite.

What is meant is the following: in classical science, space and time were always treated as the ultimate independent background variables. In some sense , space and time were "outside of space and time"! They could not be mixed up with each other, nor with matter. The equations for all other observables were in terms of these. There was a real divide

between entities existing in and of themselves, and those which were dependent on them.

One begins already to find modifications of this simple picture as far back as Galileo's principle of relative motion. Since velocity was no longer (as maintained by the Aristotelians) to be treated as an absolute quantity, it could no longer be expressed in terms of a ratio of absolute space to absolute time. One could now take the 3 quantities x,t and v, decide which pair of these to treat as independent variables, and make the remaining one dependent on them.

Newton's theory of gravitation continued this process of mixing "independent" background, and "dependent" foreground variables.. The universal gravitational constant γ could now be treated as an invariant, a constant of proportionality, within which space, time and matter were all combined. Once again, it was possible to take any 3 of the quantities x,t, v and m as independent variables, and the 4^{th} as dependent on them. Today's physics has so completely eroded the barriers between all the fundamental quantities of nature, that it is no

longer possible to state which entities are functions of others! One has, at most, the feeling that at the bottom of everything there must be a fundamental something, on which all other entities depend, but one is at a loss to state what it is or should be.

Conclusion:

The quarrel over the legitimacy of Kant's designation of Euclidean Geometry as a synthetic a priori seems to me to miss the essential point. Whatever the assumptions, structures or images a physicist choses to employ, they are designed for the purpose of creating intelligible ideas. The mathematics borrowed, invented or derived for this purpose will model these a priori assumptions, not the data itself , which has no inherent structure, nor the actual universe from which they are derived, which cannot be directly known.

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