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On Modeling Causal Singularities

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One of the major difficulties in modeling causation by analytic functions is that of modeling jump discontinuities which are *predictable* from the previous history of a system.

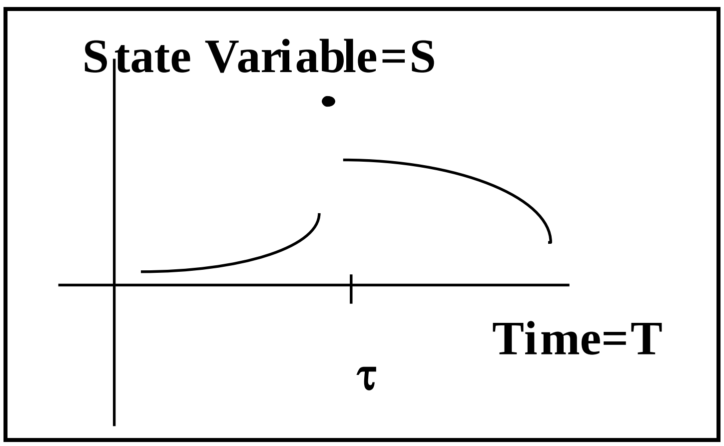


Figure I

No analytic model of the behavior of a system S before the moment t can predict the jump discontinuity at t , or the fact that S continues to function in an analytic fashion after t . However such events are a commonplace in Nature. Breakdowns, explosions, faults, ruptures, etc. are not only frequently encountered, but are predictable. In many situations it is also possible to know what will happen after the singular event has occurred. For example, a doctor may know that a patient's heart attack is imminent. He knows that it will not be fatal and that the patient, provided he follow certain procedures, will resume a normal life.

In the situations described In this article there exists a hidden process which does not reveal itself by any outward manifestations. The

external behavior of the system appears to be "smooth". The hidden process on the other hand works by slow accumulation. At certain singular moments τ_1 , τ_2 ,...τ_n ,... , it erupts in an explosion, then dies away almost as rapidly, perhaps to begin the process of accumulation once again.

In this paper we will be looking at 3 characteristic situations in analytic modeling, and interesting ways of dealing with them. Although we will not be examining the modeling of jump discontinuities by a Fourier series, these observations may readily be extended to them.

1. Rearrangements of Taylor series

Consider the infinite series representation:

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n} \quad ;0 < x \le 1$$

This series is conditionally convergent at the point x = 1. It can therefore be rearranged so that at x = 1 it will converges to any closed interval, from a discrete point to the entire real line. Choose a rearrangement ρ: n ---> n_ρ , which causes this series to converge to "3" at the point x=1. Thus:

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n_\pi} x^{n_\rho}}{n_\rho} = \begin{cases} \ln(1+x); 0 < x < 1 \\ 3; x = 1 \end{cases}$$

Let G be an analytic (real) function that rises to the value 1 at the time τ , then declines asymptotically to 0 as x --> ∞ . For example:

$$G(x) = \cos\left(\frac{\pi(x-\tau)^2}{2(1+(x-\tau)^2)}\right)$$

Let

$$\boxed{\begin{aligned} h(x) &= f(G(x)) \\ &= \ln\left(1+\cos\left(\tfrac{\pi(x-\tau)^2}{2(1+(x-\tau)^2)}\right)\right), (x \neq \tau); \\ &= 3, (x = \tau) \end{aligned}}$$

Then the behavior of h at τ *cannot* be "predicted" from its analytic representation away from t. ***However, this behavior can be predicted from knowledge of the analytic representation at any point , plus information about the rearrangement permutation , ρ .***

This permutation is the "process" . One can further imagine that this process of juggling the entries of the infinite series *proceeds in some fashion through time*. One is speaking of a "secondary time dimension s" in which it acts, invisibly, until the moment when the jump occurs. For example, if we replace G by some periodic function, say $(\sin x)^2$, and ρ by some continuous or quasi-continuous *process of continuous rearrangement* of the terms of the series for $\ln(1+ (\sin x)^2)$ one can model a situation in which the state variable S *erupts* in a quasi-random or unpredictable fashion at a predictable series of moments $x = (n+1/2)\pi$.

The process can be seen as somehow operating in the deep, secret, hidden or unknown level of the history of the state variable; something like a bridge that appears to be structurally sound until it collapses because of a systematic degradation of its strength that no-one was able to detect. Or , as Shakespeare puts it : "*This is the impostume of much wealth and peace...*"

The causal description of such systems can be symbolically represented as $F(t) = \{\phi(t); \rho(t)=s\}$. where ϕ is the smooth outward behavior and s is the "accumulator time" . Note that in the previous example, If one replaces $(\sin x)^2$ by an almost periodic function such as

$$\boxed{k(x) = A(\sin x) + B\sin(\sqrt{2}x)}$$

one can easily find ways of tweaking the argument of ln(1+x) to create a function which, in combination with the process "explodes" in quasi-random moments to random values, but which is smooth and analytic at all other times.

(2). Unknown boundary conditions

The paradigm of this situation is that of a billiard ball being deflected at the walls in its trajectory around a pool table. At any point in mid-trajectory, the state variables of the billiard ball can tell us nothing about the shape of the table at its boundaries. These state variables can give us at most mass, position, time, momentum and energy. Furthermore, if the walls of the table were to suddenly change their configuration, this alteration in the state of the "universe" would not register in any of the state variables determining the calculation of the world line of the billiard ball.

In the same way, even if, in some theoretical sense, the relative positions, masses and velocities of every particle in the universe at a given instant were known, we would know nothing about the shape of the universe in which they must interact: is it finite or infinite? Elliptic, Euclidean or Hyperbolic? Perfectly spherical or shaped in some odd way.

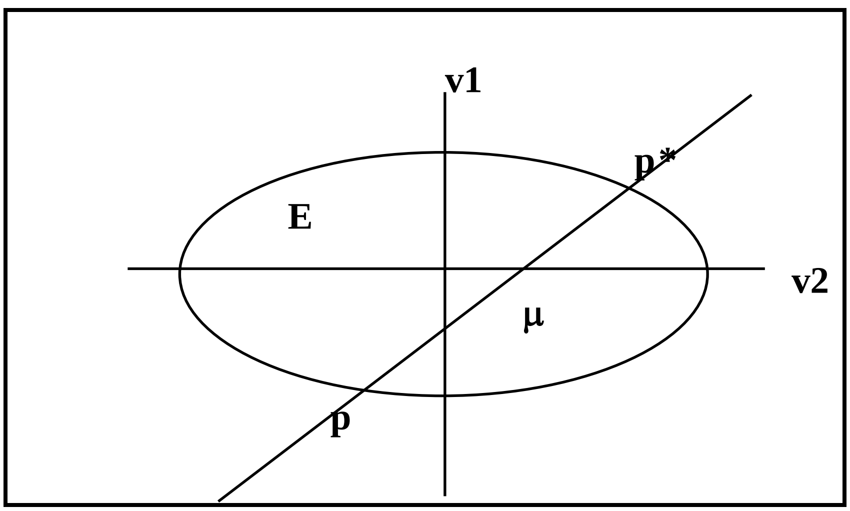
(3)Collisions

The simplest situation in mechanics, that of colliding massive objects, can be modeled by a pair of analytic equations which, together, embed 'jump discontinuity' at the instants of collision. One is speaking indeed of an *analytic variety* of collision moments

The equations are those of Energy and Momentum:

$$
\begin{array}{|l|}
\hline
E = \frac{1}{2} M_1 |v_1|^2 + \frac{1}{2} M_2 |v_2|^2 \\
\mu = M_1 v_1 + M_2 v_2 \\
\hline
\end{array}
$$

Together they define a pair of analytic curves, an ellipse and straight line, on the v_1, v_2 plane . The variety formed by the intersection of these curves is a pair of discrete points. The effect of a collision is modeled by the "jump". The velocities at $p = (v_1, v_2)$ "jump" to those at the other point $p^* = (v_1^*, v_2^*)$.



We have shown various ways by which analytic functions, and collections of analytic functions can model discontinuous and quantized behavior. To summarize:

(1) *Processes.* A conditionally converging infinite series combined with a process on the indices, can model situations in which change in hidden in the way the various contributing forces are accumulated, never revealing itself save at certain key dramatic moments of breakdown. The behavior of the system is not predictable from the function, but can be seen in the infinite series. Example:

$$y = \sum_{j=1}^{\infty} \frac{(-1)^{n_j} (\sin t)^{2n_j}}{n_j} = \ln(1 + \sin^2 t)$$

This has jumps at the points t = (n+1/2)π . If the series is systematically rearranged through permutations on the indices of the summands, it can jump to any preassigned value, or oscillate within any pre-determined segment of values $a \leq y \leq b$.

(2) ***Boundary conditions.*** **The local information derivable from the configuration of all the state variables of a billiard ball in mid-trajectory reveal only so much of its future as can be given before its next collision with the walls of the table. However, knowledge of the shape of the table combined with that of the configuration of the billiard ball, re-introduce a determinist model for all future states**

(3) Varieties of singular points obtained through the intersections of collections of analytic functions in several variables. The paradigm for this is the collision. Such discontinuous behavior does not violate causality or modelling by analytic functions.

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