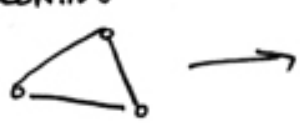
KNOTS AND POLYGONS
Roy Lisker 2011

KNOTS AND POLYGONS ①

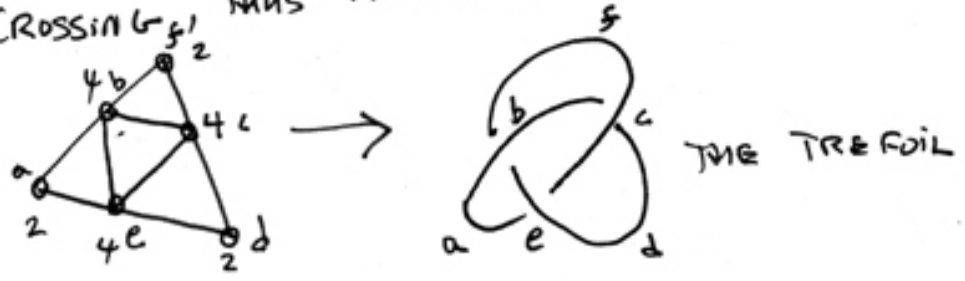
A recreation- Roy Lisker

I. LET G BE A PLANAR GRAPH. IF THE NUMBER OF EDGES AT EACH NODE IS 2 OR 4, THEN THE VERTICES CAN BE INTERPRETED AS CROSSINGS. IF THE NUMBER OF EDGES IS 2, ONE CAN SIMPLY ELIMINATE THE NODE, AS IN THE FOLLOWING EXAMPLE:



FROM THE STANDPOINT OF A KNOT DIAGRAM, THIS IS SIMPLY THE CIRCLE, LOOP, OR UNKNOT.

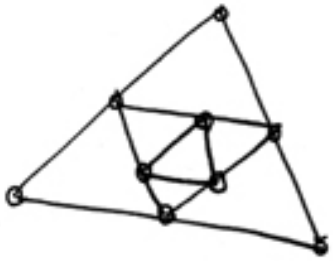
IF THE NUMBER OF EDGES IS 4, ONE CAN CONVERT THE NODE INTO A TRANSVERSAL CROSSING, THUS PRODUCING A KNOT DIAGRAM



THE TREFOIL

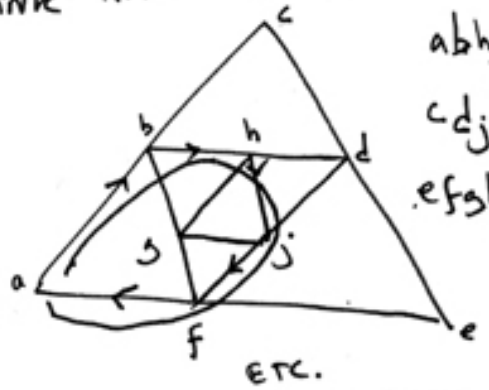
Eliminating the crossings and replacing them by nodes, converts the trefoil into a triangle intersected by an interior triangle.

Let's continue the process.



EACH 4-NODE IS CONVERTED INTO A KNOT CROSSING BY MAKING A TRANSVERSAL CONNECTION OF EDGES—

WHEN THIS PROCESS IS CARRIED OUT CONSISTENTLY, ONE DISCOVERS THAT, RATHER THAN A KNOT, ONE ENDS UP WITH A LINK WITH 3 COMPONENTS:



abhjfa

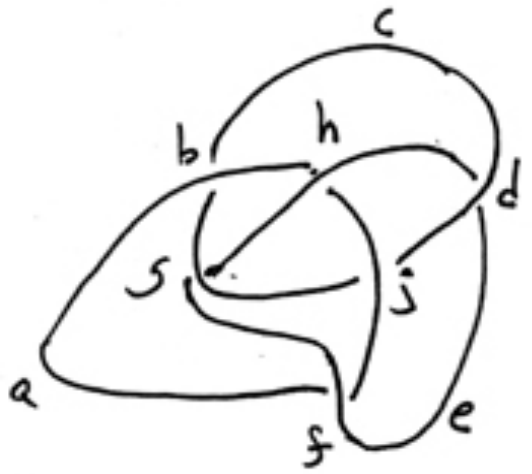
cdjgbc

efghde

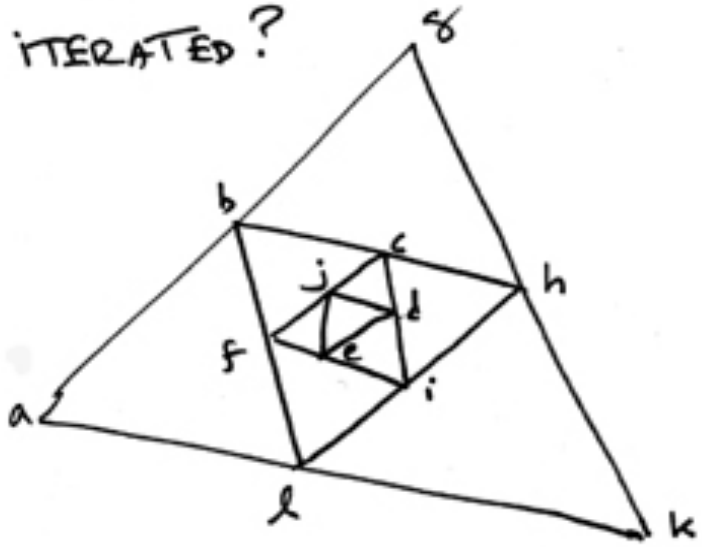
ETC.

WITH THE APPROPRIATE OVER- AND UNDER-CROSSINGS, THIS IS THE BORROMEAN RINGS DIAGRAM

Wherefore art thou, oh Borromeo!



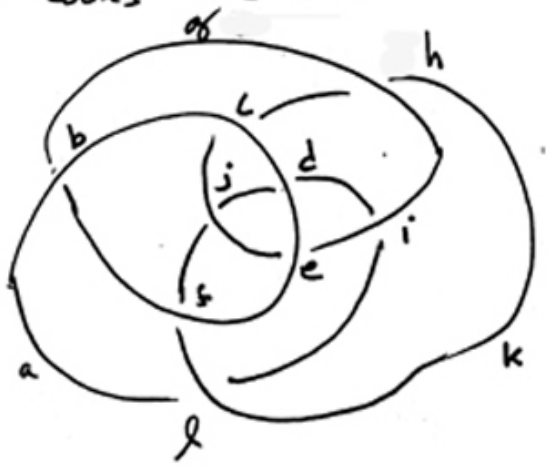
QUESTION: WHAT HAPPENS WHEN THE PROCESS IS ITERATED?



CALL THIS "STAGE 4"

THE RESULT IS, ONCE MORE, A KNOT DIAGRAM THAT MIGHT BE CALLED AN "INVOLUTED TREFOIL". SIMPLY FOLLOW THE LETTERS AROUND. THE CORRESPONDING

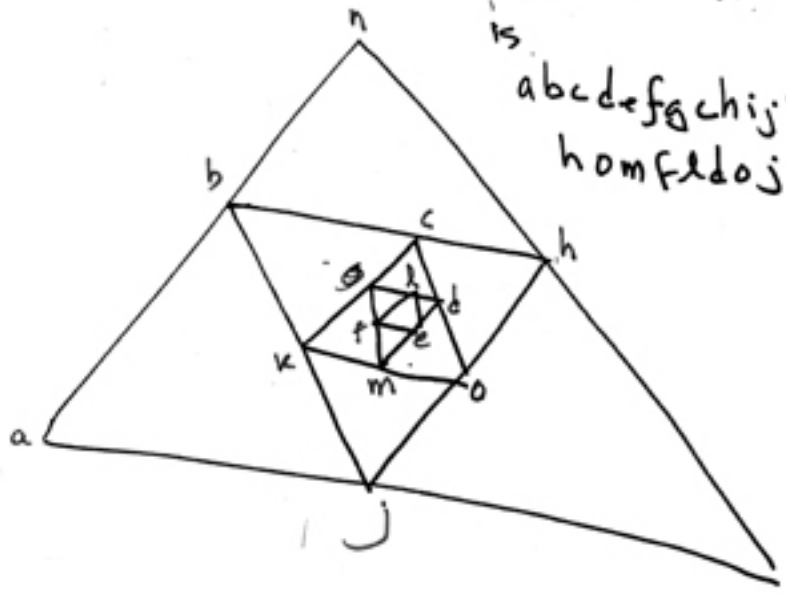
UNKNOT Looks like this:



STAGE 5

THE ORDER OF CROSSINGS IS

abcdefgchijkglemkbn homfldoja



STAGE 5 IS ANOTHER KNOT DIAGRAM!

STAGE 6

STAGE 6 IS ONCE MORE A LINK WITH 3 COMPONENTS —

What is the general rule?

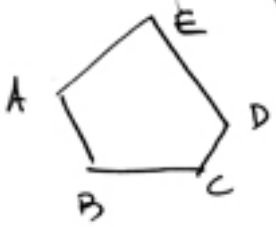
Prove it! (It's rather cute)

Observations on Knots and Polygons

II: The Pentagon

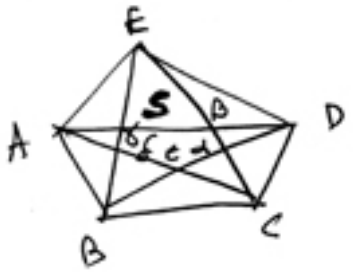
Let P be a convex pentagon

P:



Connect the vertices to form an interior star, S

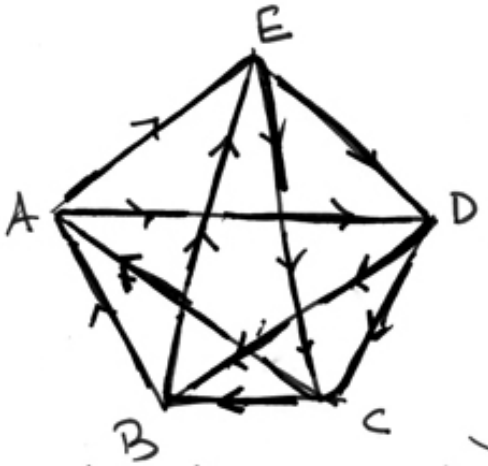
P



There are 4 lines joining each vertex. If the vertices are considered to be crossings, one has a knot diagram consisting of 5 exterior and 5 interior crossings — This may be the diagram of a knot, or the diagram of a link with several components—

This is an unknot wrapped about to form a pentagon with interior star —

(b) Alternative under- and over- crossings. This forms a knot with analogies to the trefoil



There is a freehand version of this knot.

THE "PENTAFOIL"

Observe that the pentagon is the only polygon that can be interpreted as a non-trivial knot diagram - when all the vertices are connected, only the pentagon has 4 lines meeting at a vertex, which can therefore be interpreted as a crossing