

On Containers  
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*Preface*

*On the need for a version of Set Theory  
specific to the Physical Sciences*

In setting up their construction of the integers, Russell and Whitehead define "0" as the null set, and "1" as the set containing the null set. This construction is not impervious to criticism, and we will do so elsewhere in this article. However, whether or not one accepts their interpretation, clearly that what one might call "arithmetic" entities ("0" and "1"), or philosophical abstractions ("nothing" versus "something") have to lay the ground for Mathematical Set Theory and Arithmetic. The Peano postulates, to take another example, begin with the "self evident notions" of "zero" and "successor function".

However, the axiomatics of Mathematical Set Theory , ( which we will sometimes anagrammatize ( *reference: "O Carib Isle", Hart Crane, line 5* ) to MST) is simply inadequate to deal with some very basic features of the physical universe as reflected in the physical sciences.

The problems begin with the methods for establishing the *existences* of the objects of physics, in combination with purely conceptual issues.

(1) An *element* in a set within MST is completely specified by its definition. Thus, a set containing the numbers 1.1 , 2.7 and 8.9 need not delve more deeply into the ontological foundations of these numbers. There is, of course, some flexibility here: If A is a set containing a "simple closed curve in the Euclidean plane", it is understood that one is free to choose any member of the class  $\mathcal{C}$  of all simple closed curves, what may be called a "typical representative" of  $\mathcal{C}$  .

Likewise, if one wants to take the intersection of two sets

A = {6; 2 knots; 3 closed curves}, and B = {6; 1 knot; 8 closed curves}, one does not have to worry about the possibility that the representative knots used in the depiction of A may be different from those in B, and likewise for the simple closed curves. They simply must be notated as K1, K2 and/or K3 for the knots, C1, C2, etc. ... for the closed curves. The intersection  $E = A \cap B$  will consist of the number 6, one or no "generic" knot; and 3 to no "generic" closed curves.

However, consider the following: "Let K be a set whose one element, g, is a glass G, holding a pint of water, W". g itself is a kind of set, a compound of G and W, together with a specific relationship, "containment". One might notate this as follows:  $K = \{G, W; \rho\}$ ,  $\rho$  being the relationship "containment" of W to G.

Now the arguments begin: how many impurities will be allowed into a pint of liquid before one agrees to accept it as "generic water"? What, indeed, is "generic water"? Is it a kind of

average chemical compound averaged over all pints filled with something deemed to be water? Is it a quantity of H<sub>2</sub>O with less than 5% of other chemical impurities? Or is one talking about an abstract idea known as “pure water” which exists nowhere in the universe but contains molecules holding 2 hydrogen atoms and 1 oxygen atom in an idealized state of bonding? Or should one accept a glass of pure oxygen and pure hydrogen that were bounded together in water, then separated by hydrolysis?

In a real scientific paper or lecture, “everyone” will “know what one is talking about”, when the glass of water is invoked . However, the existence of the set K is dependent on the legitimacy, by convention, observation or idealization, of G,W and  $\rho$  . In MST, the only set allowed with “non-existent” elements, is the null set.

In logical symbolism:

$$(i) \exists x \rightarrow \exists \{x\}$$

$$(ii) \neg \exists x \rightarrow \{x\} \equiv \emptyset$$

$$(iii) \exists a \wedge \exists b \wedge \dots \wedge \neg \exists z \rightarrow \{a, b, c, \dots, z\} = \emptyset$$

“(i)If x exists, then the set containing x exists.

- (ii) If  $x$  does not exist, then the set containing  $x$  is the null set.
- (iii) If  $a, b, c, \dots$  all exist, but  $z$  does not exist, then the set containing  $a, b, c, d, \dots, z$  is the null set."

Treating aggregates, collections, bounded, contained or coherent combinations of elements in this way is far too restrictive for the physical sciences (indeed for any science except mathematics) . In MST, any debate over the existence of its elements must translate into a debate over the existence of the set itself. No doubt there are quibbles about "set constructions" of the form

$S = \{\text{horse, unicorn}\}$ : "The set  $S$  contains a (generic) horse and a (generic) unicorn". Is  $S = \{\text{horse}\}$ , or the null set; or must one invoke some higher level of set theory? We will not go into such quibbles here.

Briefly, the ontology of physical objects differs sufficiently from those of mathematics, that a different version of set theory, with its own lattices and perhaps even its own logic (*c.f. the*

*quantum logics of Birkhoff, von Neumann and Reichenbach* ) is needed for their requirements.

(1) In the natural sciences, the “existence” of any identifiable object in the universe requires that it be “observable”. On the theoretical level it may be allowed if it leads to predictions of things that, once again, can be observed. An undetectable green dragon in the bedroom that regulates the room temperature is “effectively non-existent” in physics. Mixtures of visible and hypothetical objects can be assigned various degrees of “existence” depending on the theoretical context.

An excellent example is Ludwig Boltzmann’s recognition of the “usefulness” of postulating the existence of atoms. The atomic hypothesis was controversial in scientific communities all through the 19<sup>th</sup> century. In 1905 Einstein showed how one could use the atomic hypothesis to calculate the random spread of Brownian motion. Using Einstein’s equations they were detected indirectly

by Jean Perrin, then seen, much later, by the aid of the electron microscope.

This raises an interesting question, one that speaks to the concerns in this article. Boltzmann's atomic hypothesis was critical in leading to the developments that led to Einstein's work on Brownian motion, and Perrin's indirect discovery of the atom.

Question: Are Boltzmann's "atoms" the same as Perrin's "atoms"?

When one makes a statement like, "Let  $X$  be a set of 20 atoms", does one have in mind Boltzmann's or Perrin's atoms? Suppose we substitute Perrin's atoms for Boltzmann's atoms in his derivation of the H-Theorem: do they obey the Stoss-Zahl-Ansatz? The Ergodic Hypothesis? The Equipartition "Theorem"?

Otherwise stated, Boltzmann's atoms are "hypothetical objects", Perrin's were, within the limits of observation (since improved greatly) "real physical objects". The two sets  $X_1 = (20 \text{ Perrin atoms})$  and  $X_2 = (20 \text{ Boltzmann atoms})$  are as ontologically distinct as a painting by Picasso and Plato's ideal of the beautiful! They require

a different set theory, a different logic, a different Boolean lattice structure.

Ludwig Boltzmann was thoroughly justified in arguing that the atomic hypothesis was the best means available for developing a Statistical Mechanics to explain the phenomena underlying Thermodynamics. In doing so he constructed an effective “bridge” between two established branches of physics, Thermodynamics and Hamiltonian Mechanics. This bridge connected them correctly in the sense that it allowed for predictions, and because there was no better explanation. In a very real sense, then, atoms (molecules, etc.) *exist* in Statistical Mechanics, and there is no error in logic in a statement of the form, “Let  $H$  be the collection of atoms of the same constant energy in the phase space diagram of a specific gas in a specific container.”

In this context, these atoms are not unicorns, or invisible green dragons; they are back-reconstructions to entities that *are likely to*



*exist* in order to connect the results of a pair of established fields in a science.

Once again, one can argue about whether these constructions are idealizations, back reconstructions or “the best theory we have”, but what is certain is that they are not mathematical ideas, nor the kinds of “elements” that occur in MST. In fact, the *indistinguishability* of such “hypothetical” objects rule out all possibility of treating them as elements of sets, and demand that the ‘reality’ of their existence be replaced by a texture of probability densities, that is to say, pure numbers which, of course, *are* amenable to mathematical treatment.

Note in particular that, unlike the situation in mathematics, the behavior of entities singly or in small numbers (existential predicate  $\exists$ ), may be very different from their behavior in large numbers, or infinite numbers, or in their totality (universal predicate  $\forall$ ). The “local universe” is quite a bit different from the

“large scale universe”, and very different from an essentially conceptual object known as “the universe”!

(2) I’m having a conversation with someone, and I ask him

“Tell me something about your mother.”

That she is nowhere visible, that she may have died, that I may never see her in my entire life is irrelevant. My conviction of her existence is as certain as the visible presence of the person I am talking to. Even the fact that she may have only been a person who donated her ova, to be placed in another woman’s womb to become the embryo that would eventually turn into the person I’m speaking to, is irrelevant. He had to have a mother. The only purpose in pointing this out is to note that the concept of “observation” can be extended beyond things that are immediately visible. Anything “observable” *effectively exists* in physics.

More complex issues are involved in treating phenomena that one may, in some sense, prove to exist, *yet which are formally “unknowable” by virtue of the theory itself*. This might have been

considered impossible in the 19<sup>th</sup> century, but the 20<sup>th</sup> century has seen many examples of them.

Most of the examples, of course, come from the Relativities and Quantum Theory. Perhaps the simplest is this situation described by Special Relativity: if a light beam is sent into outer space, strikes an object whose velocity relative to the earth is unknown, and is bounced back, it is impossible from this alone to know where the object is, or at what time the collision took place. The reasons for this have to do with the length and time contractions of moving objects. All that one can say is that the beam did strike something at a time which (either in the reference frame of the object or of the earth), which did "exist".

In general, given any signal sent by radiation from the earth into outer space, it is impossible to know anything about its trajectory unless some independent source of information relays this back to us. In particular, if such a ray never strikes another object, it is lost to us forever.

The interior dynamics of a Black Hole is another phenomenon predicted by General Relativity about which, in theory, we can know nothing- at least if we accept the Hawking/Penrose scenario of the “trapped surface”.

The largest number of theoretically unknowable phenomena come from the Quantum Theory:

- (1) The path of the electron.
- (2) The vital status of the Schrodinger Cat
- (3) Events occurring at a scale below the Planck length
- (4) The distinguishing characteristics of the particles in a Bose-Einstein condensate
- (5) The distinguishing characteristics of the “two”(?) particles in an entangled singlet.

Can standard MST deal with “elements” defined and described in this way? Note that, even though one cannot “distinguish” the electrons in a electron gas, it can be weighted, and in theory, one can know *how many* electrons it holds. Yet

they cannot be distinguished. Therefore the statement “Let B be the set of all particles in a Bose-Einstein condensate” would appear to make little sense.

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In summary, the elements of a set in MST must meet a list of requirements that one simply cannot expect to find for many of the entities of modern physics:

(1) Their “existence” must be not ambiguous, nor subject to debate, nor provisional. In the natural sciences, on the other hand, one must allow even the possibility of the existence of unicorns, if these give the best explanation for observed data

(2) The “existence” of the elements of a mathematical set does not derive from observation, or from apparatuses or instruments used in the making of such observations

(3) The potentially infinitely ascending tower of primary

attributes supporting secondary supporting tertiary supporting n-level attributes, can have no place in the description of the elements of mathematical set theory

(4) Above all, the elements of a mathematical set must be distinguishable. This does not prevail for a great many of the entities in modern physical theory.

### *Containers*

Any dispute about the “existence” of the elements of a mathematical set inevitably implies a dispute about the “existence” of the set itself. The notion propounded here of a *container* is designed to circumvent this: A container captures the idea of a collective aggregate, like a set; however the existence of the containing entity does not depend on the existence of its individual elements. This must be understood as a first attempt only towards untangling the “ontological nightmare” that plagues so modern physics, and, perhaps, will always be with us.

## *I. The Ontological dilemmas of the Set Concept*

The principal logical failing in the set concept, (as elaborated for example by the Zermelo-Fraenkel or axioms), is that the existence of a set,  $S$ , is dependent on the existences of its elements. These elements are assumed, by various arguments having nothing to do with their assemblage as a set, and at least as a hypothesis (in which case we are talking about hypothetical sets) to have prior existence.

It is also assumed that these prior existents have distinguished existence. Each one is distinct and different from the other.

Given distinguished elements  $e, f, g, \dots$  etc., the assertion of their "existence" may be notated as  $(\exists e \wedge \exists f \wedge \exists g$  From this one makes use of the

Set Postulate:  $(i) \exists e \rightarrow \exists \{e\}$

*"The existence of  $e$  implies the existence of the set containing  $e$ ."*

If  $S$  is a well-founded set, and  $e$  an entity whose existence has been asserted by various means, then the union set  $S'$  comprising the contents of  $S$  with the addition of  $e$  is guaranteed by ZF.

Note that:

(1) Sets must have elements. We will examine the special status of the null set in a moment.

(2) These elements must be distinguishable.

There are several ways of looking at the vacuous set that bring it into conformity to these statements. We assume, as is often done in MST, that there is a "universe" of elements  $U$ , from which all well-founded sets derive their existence. The "existence" of all the elements in  $U$  implies by the Set Postulate, the existence of  $W = \{U\}$ , the set containing  $U$ . This once again, has only a dependent existence on the existence of its elements. The "vacuous set"  $\phi$  can then be defined as the complement of  $W$ . If some or all of the elements of  $U$  do not exist, then the "vacuous set relative to the



universe  $U$ ” also does not exist. This sidesteps the problem of whether the set of all unicorns is the same, or different from the vacuous set. In the way that we use the concept of a container, we will argue that it makes more sense to speak of the “vacuous container”, than of the “vacuous set”

At any rate, if  $S$  is any set, then the addition of a new element,  $f$ , to the content of  $S$  destroys the “existence” of  $S$ . It is playing a bit of sophistry to argue that if an element could be removed from the vacuous set it would cease to exist, but because this can’t be done, it must exist!

In our view it is a serious error in logic to deduce, from the fact that the null set has no content, that it somehow exists apart from its content (which doesn’t exist)! In the same way, it is our content that the construction by Russell and Whitehead of the integers should not begin with  $1 = \{\phi\}$ , that is “1 is the set that contains the set that contains nothing, but rather a postulate to the effect that

*“since something quantifiable (whether in the universe or thought) exists, we will call a standard quantity of that something “1”.*

This is consonant with the way the “unit” normally works in the world of magnitudes: one selects a certain weight, the ounce for example, and designates as “1” the weight of a mass that weighs one ounce. This operation is not possible if one wishes to go from the null set to the “unit set”

In summary: since the existences of all sets except the “null set” are dependent on their contents, we do not see why any exception should be made for the null set.

## *II. The need for an axiomatics of “Containers”*

The above discussion might be dismissed as academic quibbling over the fine points of definitions in logic. Our focus in this preliminary article however, is on the *ontological nightmares of modern physics* , a state of confusion between observation and

logic, between science and philosophy, that is unprecedented in scientific history.

In modern physics one finds many concepts which possess the attributes of "containment" which find no equivalent in classical set theory (nor, for that matter, in traditional geometries.) The degree to which a "container" and its "content" can or cannot be separated differs from one branch of physics to the next.

Examples proliferate: the presence or absence of a space-time *background* issue in General Relativity; the phenomenon of *quantum tunneling* ; the *locality paradoxes* of Quantum Optics; the slipperiness of the *entropy concept* , particularly for systems out of equilibrium; the ubiquitous *dark matter; dark energy; etc.*

The first requirement for our definition of a container will be that it somehow *exist* (in some reasonable use of the concept) , *independently* of its contents. Let's (provisionally) notate "containment" by the symbol  $[( )]_{\alpha}$ . The index " $\alpha$ " specifies which container is under consideration. This already highlights the fact

that it exists apart from its contents. (Unless necessary, we won't notate the index.) Then its ontological attributes must include :

(1) The existence of "x" does *not* imply that x is (or even can be)

the contents of a container:  $\neg(\exists\{x\} \rightarrow \exists[(x)])$

(2) Containers should possess what may be called a "vacuous state": in some sense they "exist" independently of any contents (whose 'existence' is justified by other protocols, observation,

logic, definition, etc:  $\exists([() ]_\alpha) \neg \rightarrow \exists x \text{ such that } ([ (x) ]_\alpha)$

One can loosen up this requirement for entities that may be considered midway between containers and sets: they must contain something taken from a list of potential contents.

(3) The potential contents of a container K may be called its "states",  $\sigma$ . If L is the list of states, then the full description of a container may be notated  $K = (K_0; L; \sigma)$  Here  $K_0$  refers to the "vacuous state", if it applies: wine bottles can exist without wine.

(4) The existence of a container is contingent, neither on the existence of the container, nor of a specific state of that container .

In this preliminary paper we will be looking at 3 classic examples of containers:

(1) Boundaries; Fields; Potential Wells

(2) Continua

(3) Observables

### *III. Boundaries : Venn Diagrams and the Jordan Curve Theorem*

Let  $L$  be a simple closed curve in the Euclidean Plane. By the Jordan Curve Theorem,  $L$  separates the plane into (1) an "inside" (2) an "outside" (3)  $L$  itself .

If " $L$ " is lifted isometrically into 3-space it loses its capacity for "containment". Thus  $L$  is a boundary, a certain kind of container, not a set. Indeed, if one designates the copy of  $L$  in space as  $L'$ , and projects  $L'$  onto  $L$  in the plane, we see that  $L'$  exists apart from any properties it might have of containment, and that

these properties only become manifest when projected onto a plane surface.

At the same time, the “set” consisting all locations on the places “inside”  $L$  disappears by removing a single point inside  $L$ !

Venn diagrams are most effective as tools in MST if they are thought of as convex regions, or even as a collection of disconnected convex regions : One starts with two finite set of points, or locations  $Q = \{q_1, q_2, \dots, q_n\}$ ,  $P = \{p_1, p_2, \dots, p_n\}$  (easily generalized to infinitely many) , in the plane. One then draws a boundary  $A$ , that encloses an entire region containing  $P$ , yet leaves the set “ $Q$ ” on the outside.  $A$  is a “boundary”; there is no notion that the sets  $P$  and  $Q$  somehow “create the inside “ $I$ ” of  $A$ , or its complement  $J = I^c$ .

As long as we stick to these conventions, we can bring in the full Boolean lattice of Set Theory and speak of “unions”, “intersections” , “complements” and so on.

#### *IV. The Axiom of Choice*

In some sense, the Axiom of Choice is *implicit* in MST  
by virtue of the 2 ontological requirements of a set:

(1) The existence of a set  $S$  is *dependent* on the existence  
of its elements

(2) All of the elements of  $S$  are *distinguished* .

Because of these requirements, one can in theory *select out any element by virtue of its being unlike any other element of  $S$* . This leads, (intuitively if not strictly logically) to the procedure known as “counting”, and, with a few more convenient assumptions, the Well-Ordering Theorem.

At the same time we know of “countable sets” that cannot be counted. For example, the set  $K$  of all computable numbers on the real line. Arguments equivalent to the constructions in Gödel’s Theorem show that  $K$  is indeed “countable”, yet, using a simple Cantor diagonalization, there is no algorithm that can be constructed to do the “counting” (something which is not true for the rationals for example). Unambiguous counting thus serves as a

kind of touchstone, a line of demarcation between the set concept, and other ways of designating a collectivity.

To our way of thinking the Axiom of Choice, particularly with respects to the objects and concepts of the physical science, should be allowed to vary as a function of the specific context. Phenomena such as entanglement, non-locality, Black Holes, even turbulence and statistical mechanics, do not comfortably admit the notion that it is desirable, or even possible, to select out individuals from aggregates of entities, whether they be particles, fields, continua, uncertain locations, etc.

In Statistical Mechanics, for example, one is confronted by situations that one can characterize as *partial indistinguishability*. The phase space of a dynamical system is decomposed into tiny boxes, so small in fact that one cannot say how many particles are in them, only that the box is empty or full. Each box holding one or more particles is given the weight "1"; the empty boxes have weight "0". By thus relaxing of the notion of "choice" one is able



to define a “fictive density”, very unlike the Liouville density which is an invariant constant, from which one derives Boltzmann’s equivalence of entropy with the logarithm of this ‘semi-mathematical’ phase space volume.

### *The Axiomatics of Container Theory*

Our conception of a “Container” therefore begins with these exceptions to the classical Zermelo-Fraenkel axioms of Set Theory:

(1) The existence of a container *is* not dependent on its content.

(2) The elements of the content of a container need not be distinct. This will become clearer when we discuss continua, observables, potentials, fields, frames and spaces.

(3) The Axiom of Choice may be weakened, or dispensed with altogether

(4) A container may contain itself. Here I am motivated primarily by ideas from Aesthetics. It is often the case that, in works of art, notably music, that the form can be treated as an element of the content.

Reviewing each of these conditions in turn:

(1) The assertion  $\sigma = \{a_1, \dots, a_n\} \rightarrow \exists[(\sigma)]$

is not an essential property of a container: the state  $\sigma$ , consisting of a set of numbers  $a_1, \dots, a_n$ , (for example) must be established independently from the existence of the container  $[\sigma]$ .

(2) Let L be the "list" of "states" ( $\sigma$ ), that is to say, "permissible contents" of a given container J. Then

(a) L may simply be an inventory of sets, a kind of databank

(b) A container, J, can be defined *autonomously*, setting up the list of potential states L by implication. More fundamental than this is the property:

(c) Containers can have "open" lists, for which there may not exist a complete inventory of permissible contents. One example of such is the container word "Space".

(d) One of the things that the container concept may be able to

address is the dubious status of statements in standard set theory of the form: "*Let H be the set of all the birds in Pennsylvania on the morning of June 10,1976*". The problem with this seemingly innocuous definition is the following:

The existence of the set H is dependent on the existence of its elements. However, there is no way of knowing if any method of detection can give a definite number to all the birds in Pennsylvania on that date. One can always argue that some birds went undetected. The statement is very different from "*Let H be the set of all birds observed (by some method) on a certain date*".

*In other words, an ontologically dependent concept, the "setness" of H, relies on an entity "ALL birds" that is intrinsically unknowable.*

The idea of an "open container" allows for such things: Pennsylvania is a region on earth in which there are birds. We know that it always contains at least one bird, and probably others. Thus the "set" of all birds there on a certain date, is *contained in the*

*boundary container "Pennsylvania"* , which does not depend for its existence on the existence of birds. One can state this as a tautology:

*The container "Pennsylvania ( at a certain date) contains all the birds contained in Pennsylvania"!*

Since the existence of Pennsylvania does not depend on its birds (unless one is an ecologist!) , one avoids the usual quibbles about sets.

Note how the phenomenology of "observation", the most controversial issue in modern physics, is the issue in setting up this paradox in Set Theory, which may be (partially resolved) by the introduction of containers.

## *V. Continua.*

(1) Before the work of Dedekind and Cantor, it was not customary to think of a continuum as something that could be decomposed into distinct locations. Granting that the real line, as analysed by mathematicians of the 19<sup>th</sup> century, is the prime

example of a *decomposable* continuum, decomposability is not a *prima fascia* property of a continuum. It has not been customary in the history of science to assume that, for example, liquids , gases and solids were decomposable as a gigantic, potentially infinite number of point-objects.

Democritus, and the Epicurian school as exemplified most brilliantly by Lucretius, argued that the visible objects in the world of daily appearances could be reduced to collections of atoms. However, following Lucretius, they assumed that the atoms themselves were hard spheres, sometimes with “hooks” and with a tendency to “swerve” in their downward paths, yet *in themselves* indivisible.

When Ludwig Boltzmann attempted to lay the foundations of Statistical Mechanics by the depiction of gases as aggregates of atoms or molecules, he did not maintain that these existed in the real world. In fact, Boltzmann constantly changed his pictures of

sub-microscopic reality to fit his mathematics, depicting these elementary particles as

- (1) hard spheres
- (2) point particles
- (3) indivisible continua; or probability distributions.

*For us, the defining condition of a continuum is in the connectedness of all of its parts; note that we do not use the term "elements".*

Watching the flow of a river , the lava from a volcano; listening to an extended sound in a musical composition; even when thinking of the flow of time, it is not customary to try to dissect these experiences into point locations. Even in the customary notion of the "instant", there is a tripartite division of emerging from the past, verifying the experience of the present (which takes time), moving into the future. What is considered to characterize a continuum is:

- (1) A continuum has parts
- (2) These "parts" are themselves sub-continua

(3) One can speak of closer and more distant parts relative to a given part.

(4) There is no discernible “line of demarcation” between a part and its neighbors. This condition can be loosened as required.

(5) It is possible to “travel” from one “part” of a continuum to any other by a connected route involving neighboring parts.

The real line  $\mathbb{R}$ , is a continuum in this sense. That it is decomposable is an added benefit. An open covering of a segment on the real line is a continuum if the ‘indiscernible boundary’ condition is loosened.

One can add a few more conditions, corresponding to the common notions of the composition of a “fluid”

(6) In keeping with the notion of “substance” whether solid, liquid or gas, one must in fact *abolish* the possibility of any sort of reduction to fixed point particles or locations. Any form of substance must occupy a positive volume; “Euclidean points” may exist in mathematics, but they are foreign to the whole spirit of

physics, and we are looking for concepts that are relevant to physics. For example, the density of a particle reduced to a point, is  $0/0$  ! A “zero amount of substance” is not conventionally considered to be substantial.

(7) One of the consequences of (6) is that the complement of a part is also a continuum. To express this requires two notions

a. If P is a “part” of a continuum C, there exists a “part” Q which is disjoint from P:  $P \cap Q$  is vacuous

b. Let R be the union of all the parts of C disjoint from P.

Then this is either a continuum or a union of continua.

## *VI Observables*

In the context of this article, an “observable” is a conceptualized entity, either a mathematical construct or some entity abstracted from physical observation and theory.

(i) It has no distinguishable locations, elements, points or parts

(ii) One knows *all that is possible to know about it* through



the *measurements of characteristic magnitudes or quantities*. These measurements can be made, abstractly or with physical instruments, producing a list of numbers (real, complex, elements in some Lie algebra, etc.) that constitute its' "state",  $\sigma$ .

(iii) The list of all possible states constitute the *potential or capacity* of the observable as container.

Examples of such measurable quantities are of course, weight, energy, space-time location, spin, volume, density, number, etc.

### *Time, the Paradigm of Observables*

Consider the manner in which a state of the observable "time" is measured by a clock. A clock is a dynamic system in a space-time region of "acceptable isolation" ( *John Hamilton, inventor of the chronometer, devoted his entire life to creating such "conditions of acceptable isolation".* ). Its' defining property is that it exactly reproduces at some future time  $t_1 = t_0 + \pi$ , (within a controlled margin of error), an *initial state*  $\Sigma$  at time  $t_0$ .

$\pi$  is its period. Under the assumption of strict causality, this initial state will be reproduced, at exact intervals  $\pi$ , indefinitely, (or, more realistically, until one of its moving parts runs down.)

What does one mean by “the initial state of a clock”? It is a state of a very great number, potentially infinite list of numbers describing all its moving parts, (with generic representative  $p$ ), through its list of time derivative derivatives,  $dp/dt$ ,  $dp^2/d^2t$ ,  $dp^3/d^3t$  .....

This list is the Observable for the clock; the Observable for the magnitude “time”, is just the period  $\pi$ , and the initial time, which can be taken to be “0”. *Just as in quantum theory, the state of the measuring apparatus must be included in the state description, as well as the magnitude being measured.*

The State itself is just a collection of numbers and can therefore be treated as a set. The Observable itself cannot be comfortably described as a set, but fits well within the concept of a container.

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### *Summary*

A dispute over the existence of the elements of any set translates into an ontological dispute about the existence of the set itself. In physics in particular, the many disputes about the “existence” of just about everything questions the legitimacy of any set comprised of these objects.

Furthermore there is a large list of phenomena in physics which possess only a contingent existence, based on a causal chain that, hopefully, leads back to some fundamental entity that does not owe its “hypothesized existence” to anything else. Or, at least, there is a general consensus that the indefinite descent can be terminated at that point.

It is because of this, that physics (and the natural sciences in general) is in need of alternatives to Mathematical Set Theory (thus, to Mathematical Logic as well, its close cousin). “Physical sets” , existing apart from their content, may be the way to allow

for the legitimate treatment of collections of elements whose ontological status are disputed, either singly or in the aggregate.

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