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On Black Holes and the unification of physics.

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A number of isolated observations can be combined to produce an interesting, and perhaps original, conclusion. The first of these is an important result from Differential Topology, known as Sard's Theorem. From its succinct statement in Wikipedia:

"Sard's theorem, also known as Sard's lemma or the Morse–Sard theorem, is a result in mathematical analysis which asserts that the image of the set of critical points of a smooth function f from one Euclidean space or manifold to another has Lebesgue measure 0 – they form a null set. This makes it "small" in the sense of a generic property."

This result was applied by Rene Thom to the manifold of points he called the Catastrophe Set: those places during some process or dynamical system, (representable by a manifold M in some Euclidean space), of dimension k and co-dimension n-k, in which there is a collapse or reduction in dimension into one or more of his 7 basic or elementary "catastrophes".

Thom's insight places a high scientific value on the manifolds of singularities:

(1) In agreement with Sard's Theorem, the catastrophe manifold, the embedding into the tangent plane, of the locus of all the singularities is of lower dimension than the parametrized configuration space $M = C \otimes X$ over which it is embedded. To quote the generally reliable Wikipedia:

"Catastrophe theory analyses degenerate critical points of the potential function — points where not just the first derivative, but one or more higher derivatives of the potential function are also zero. These are called the germs of the catastrophe geometries. The degeneracy of these critical points can be unfolded by expanding the potential function as a Taylor series in small perturbations of the parameters.

When the degenerate points are not merely accidental, but are structurally stable, the degenerate points exist as organising centres for particular geometric structures of lower degeneracy, with critical features in the parameter space around them.

(2) Next: is precisely on this singular manifold (which is very often no more than a collection of discrete points), that all the interesting, non-trivial topology is located. Otherwise stated, the

catastrophe or singular set contains all of the information about the topological structure of the manifold, $M = C \otimes X$.

(2) It is universally recognized that Quantum Theory and General Relativity are irreconcilable. Their combination leads to contradictions whenever they come together outside the domain of the exceedingly tiny (Quantum Theory), or the very large and very fast (General and Special Relativity).

To this dilemma one can add Statistical Mechanics as well. For, although one is encouraged (by the anxiety-prone) to entertain the fiction that Statistical Mechanics is really Classical Mechanics in the case in which one is dealing with a system of a huge number of tiny molecules, in fact when one comes to the lower end of the scale, near Absolute Zero, the notions of Entropy (Thermodynamics) and Zero-Point Energy (Quantum Theory), fade into each other in a way that is paradoxical and not understood. This is the essence of Nernst's Third Law of Thermodynamics.

Referring once again to Wikipedia:

"The most common enunciation of the third law of thermodynamics is: As a system approaches absolute zero, all processes cease and the entropy of the system approaches a minimum value. This minimum value, the residual entropy, is not necessarily zero, although it is

always zero for a perfect crystal in which there is only one possible ground state.

(3) Consider what is known, believed or speculated about the structure of Black Holes:

(a) A Black Hole is a singularity. This already has two interpretations: it can be a singularity in Space-Time, or it can be a point that has been "cut out" of the Space-Time manifold. In both interpretations it is a place at which the manifold ceases to be smooth and develops a complex topology.

(b) It follows that all of the non-trivial topology of Space-Time is contained in Black Holes, the Big Bang and other singularities arising from the solutions of the Einstein Field Equations.

(c) The Black Hole is an object that unifies General Relativity and Quantum Theory. The phenomenon of Hawking Radiation which depends on both theories, also brings into play Statistical Mechanics and Thermodynamics. The "surface temperature" of a Black Hole is a direct generalization, perhaps the most complete context within which ordinary temperature can be defined.

Quoting again from that same universal fount of wisdom:

"The only way to satisfy the second law of thermodynamics is to admit that black holes have entropy. If black holes carried no entropy, it would be possible to violate the second law by throwing mass into the black hole. The increase of the entropy of the black hole more than compensates for the decrease of the entropy carried by the object that was swallowed."

(d) Thus, the 3 modern theories of the universe find their reconciliation in a non-physical singularity at which the normal laws of causation break down. This is not surprising, and reflects Thom's observation that it is at the singularities of a mapping between manifolds, namely the Catastrophe Manifold, that all of the relevant structural information is to be found.

(5) Finally, one can argue that *it makes perfect sense* that General Relativity, Quantum Theory and Statistical Mechanics should be starkly at odds on all the 'regular points" of space-time, and only come together properly on the singularities that we call Black Holes.