Non-Metrizable Time

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Any dynamical system K in isolation from external forces carries, along with the "local" or "universal" time of the surrounding universe, an intrinsic time dimension, inherent in the characteristic properties of its own dynamics. If it is impossible to construct a clock within K, we will say that its time is non-metrizable.

The distinction between "metrizable" and "non-metrizable" time can be illustrated by means of examples taken from common experience.

Example 1: **Let S be the molecules in a glass of water at a room temperature held constant, under the action of Brownian Motion. Imagine an instrument Q, that can locate the positions of each molecule at a given instant in time.**

Q takes readings at one minute intervals. It does this for several days, accumulating thousands of pages of data which are put aside for analysis.

Case I: **The order of the pages is maintained. The configurations on each page are clearly drawn. However one is unable trace the path of any molecule from one page to the next.**

In this case there is no way to build an internal clock to measure the passage of time. The relationship of each of the pages to one another is totally random, and no cyclic or uniform pattern

of motion can be discerned. This is an example of non-metrizable time. As Mandelbrot explains in *The Fractal Geometry of Nature,* (pg. 236: All references are to the Bibliography):

" The Brown trail is creaseless".. (The 'Brown trail' is the one-dimensional analogue to the stack of measurements in storage described in my model) .." Given an interval corresponding to the time span t, one cannot tell the span's position along the x-axis. (Italics added) Probabilists say that a Brown trail has 'stationary increments.' "

Case II : It is somehow possible to keep track of each molecule from one page to the next. In this case one can use the Einstein-Smolochowski relation which relates displacement to time, to construct a statistical clock for system K. Under these conditions, the system is metrizable, though only approximately.

The internal time should be distinguished from the "stop-watch" mechanism that was used for taking "pictures" K. This time is *outside* the system and does not interact with it. Quoting once more from Mandelbrot, op. cit., pg. 235:

"Scaling by t is characteristic of most aspects of Brownian motion. For example, the distance it covers in time t, measured as the crow flies, is a random multiple of t. Also, the total time spent in a circle of radius R around B (0) = 0, is a random multiple of R^2*."*

Example 2: Stationary Universes:

The simplest example of a universe without an internal time dimension is just a solid rigid massive particle at rest in one's reference frame. Lacking internal time, it lacks as well any kind of internal subsystem that can function as a clock for measuring time.

In classical physics from Aristotle up to an including Newton, the same properties were ascribed to the fixed stars which, since they did not and would never move, were outside of time and thus "timeless".

If the particle moves in a Galilean " absolute time" reference frame at uniform velocity, then one may use it as the functioning mechanism of a clock, in the space of the observer. However, the subuniverse defined by the reference frame of the particle itself will lack a time dimension, the particle being at rest. In such a situation, the question of the metrizability of time is moot. Without motion there can be no time.

Another kind of stationary time is present in the configuration of a body B heated to and maintained at a fixed constant temperature. If there is temperature there must be motion, and if there is motion, there must be time. Yet, although time is measured by the accumulation of a quantity of motion, there is no way by which such an "accumulation" can be effected without the conversion of some of the heat in B to an equivalent amount W of work which can be used to run a clock C . Such a situation is no longer stationary.

This example shows that it is possible to conceive of a universe in which time exists, but cannot be measured. In order to build C, one must step outside the restrictions of the state of the body B, into the dynamics of the larger universe that surrounds it.

Example 3: **S is a two-body system, a simplified abstraction of the movement of the Earth orbiting about the Sun with all perturbing influences removed.**

To a first approximation the Earth passes the location of the vernal equinox on its orbit at the same moment of each year. Provided that there exists some unambiguous way of identifying the equinoctial location, and discounting perturbations, S functions as a clock; its period is the year. Without modification it can't measure time intervals of duration less than a year. An (idealized) Moon rotating about the Earth and an (idealized) rotation of the Earth about its axis can be added to this model to create a system S*, which permits this. As shown in my paper *Euclidean Time and Relativity, (*see Bibliography), *this collection of 3 clocks cannot measure time intervals less than the minimum period among them*, which in this case is the rotation of the earth. This is because, time being irreversible, clocks can't be coupled in the way rulers can, to construct a period based on *differences* between periods of distinct clocks. See the discussion further on.

The intrinsic time dimensions of S and S* are *metrizable,* within a margin of error. This margin has important consequences in our own world when seeking confirmation of Kepler's Laws, and in establishing the official duration of the second.

This lengthy quotation is from Audoin and Guinot, *The Measurement of Time.* , pg. 48 (translated from an article by *A. Danjon* in *L'Astronomie, 1929*; 43, 13-22):

'It's legitimate to consider the rotation of the Earth as the cause for the apparent disorder that still reigns in the Solar System. Although Newton's law has been saved, it is experiencing a quite extraordinary adventure: henceforth called up to gauge the passage of time, it becomes in part unverifiable and ceases to be what could strictly be termed a law [...] Since we would ask

[Kepler's] laws to provide a measure for the passage of time, we could no longer subject them to experimental control without entering into a vicious circle [...]. Let us simply hope we shall one day discover a good terrestrial time standard, so that we may leave these purely logical difficulties behind us."

These "logical difficulties" were not resolved until the introduction of atomic clocks in 1955 (op. cit., pgs. 48-49).

Other examples of both metrizable and non-metrizable time continua will be presented in this paper. They will show that both metrizable and non-metrizable systems exist within the normal functioning of our own universe. Such systems, considered as existing in isolation from all external forces, can be treated as autonomous universes.

One is naturally led to investigate universe models with metrizable or non-metrizable internal time dimensions. A question inevitably arises: is the time continuum of our own universe metrizable or non-metrizable? This article doesn't attempt to answer this question. The arguments presented here do however question the usual assumption that the existence of a time dimension implies that it is automatically quantifiable.

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The astonishing success of the Theory of Relativity has encouraged the physics community to think about "time" almost exclusively in terms of one model only, the real line R, or linear spatial continuum. Most people are quite happy to totally identify local time with the real line. Any other way of looking at time is deemed absurd at best or cranky at worst.

But the "real line" is not "time", it is just a model for a certain way of looking at time. The quantification of time should not be confused with time itself. The temporal continuum has been assumed to be quantifiable since the 17th century. No one can doubt that this perspective has been extremely useful for casting Newtonian mechanics, Hamilton- Lagrange dynamics and related other physical theories into forms that can be used for making corroborating predictions, which is what science should be doing.

Special and General Relativity have gone further. Not only is time an affine parameter, but it is actually a geometric dimension in a non-Euclidean metric space, a Minkowskian or Riemannian space-time. Minkowski space is very specific, but Riemannian space-times are quite general. Fixing a certain location in 3-space as origin, (the ground beneath our feet for example) , the trajectory of the "world line" fixed at that origin and going forwards and backwards in time alone, should be a 1-dimensional manifold embedded in a 4-dimensional manifold $\mathbf{M}$. It therefore has to have a topology, with a clearly defined Poincaré group or fundamental group of homotopies.

There is a classic theorem which tells us that one can impose a Riemannian metric on any smooth manifold. Quote Albert Schwarz, *Topology for Physicists* , pg. 176:

"One can show that any fibration whose fiber is contractible has a section. In fact, any fibration whose base space is k-dimensional and whose fiber is aspherical in dimensions less than k, has a section [...] This often leads to important information.

Let's show, for example, that every smooth manifold M has a Riemannian metric ..."

Once the topology is given, a smooth manifold can always be turned, locally, into a metric space. The Riemannian metric is a covariant differential two-form with which one should be able to make local measurements, if not always global ones.

Yet "time "does not work that way! **Numerous arguments will be presented in this paper to show that, when dealing with a temporal continuum, one must know the metric** *first* **in order to determine the topology.**

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The epistemological differences between "length" and "duration" are not merely issues of philosophy. They translate into the kinds of mathematics one uses in describing them, and the physical systems used to measure them. Here are some of the more significant differences:

A. *Irreversibility:* **This epistemological constraint does not necessarily figure into a mathematical model. It is really a description of our ability to** *know* **or recapture events in past time, and isn't concerned with the philosophical status of the** *existence* **of past events. "Earlier" and "later" are qualifiers of** *perceptions* **and** *judgments*. **Since we have good reason to believe that the Battle of Hastings really happened, we do not dispute its "existence". Wormholes aside, one normally discounts the ability of conscious minds to return to it as witnesses, (or participants!)**

This point of view has pragmatic consequences for us: ***we will define the forward direction of time, as that in which the act of measuring temporal duration is performed.***

In measuring a duration, there must be an initial moment, an event Mi and a terminal moment or event, Mt. We assume that initial observations of clock configurations always come ***before*** **terminal observations. The intervening duration is also an event which may be called the** ***waiting period.***

When the initial measurement is made, it is assumed that a waiting period will intervene before the terminal measurement can be made. When examining the configuration of the clock at the terminal moment it is impossible to redo the initial measurement.

For science, conceived of as a human activity, the division of time into past, present and future has major consequences:

1. The configuration of a ***past event*** **, or past world-picture must be** ***deduced*** **from a combination of present features (including memories). Calculations of parameters of past events are based on the** ***derived*** **laws of nature: it is our sorry fate on Earth that these laws are not given to us in advance, but have had to be uncovered with incredible labor over many centuries.**

Our "knowledge" of the past is at best a mathematical abstraction, normally a set of conjectures or hypotheses forever in need of supporting arguments and new evidences.

A major corollary of this fact is that any reconstruction of the past from the present which is free from logical contradictions or mathematical inconsistencies, is as valid as any other reconstruction. If two quite different causal chains are developed

that establish the provenance of a present event, and if the evidence to establish priority for one or the other is absent , or if there never was any evidence, both images of the past must be accepted as equally correct.

These collections of alternative past reconstructions are important when dating historical artifacts. At the present moment there is a fierce dispute raging between the Egyptologists and the geologists. The Egyptologists assert that all the evidences coming from archaeology and the historical record, show that the Sphinx was constructed 4500 years ago. The geologists, based on terrestrial features which point unmistakably to erosion by rainfall and inundation, place the building of the Sphinx as far back as 10,000 years in the past.

No doubt there will eventually be a reconciliation of these viewpoints. For the present, however, one is in a situation not dissimilar to that of the Schrödinger cat paradox, whereby a dead and a living cat must assumed to be simultaneously real.

2. For the purposes of science, the *present* is "validated" by observation, inspection and experiment. It is not "deduced" from equations or chains of causal reasoning. In this article "making an observation" and "being in the present" will be considered equivalent statements.

3. Knowledge of the "future" combines features of (1) and (2). Knowledge acquired in the present, combined with derived laws, is used to *fabricate* an image, a hypothesis, of some future moment. This image will be compared with the events at that moment when it does arrive. A "double image" (x, y) combining (x), the

prediction based on hypotheses, with the future event y is foreign to the ways in which both the past and the present are considered.

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Unlike duration, spatial length, (and its generalizations to n-space) cannot be decomposed into past, present and future. The "selection" of an origin, on a line, or in 3-space, (or n-space), is completely arbitrary. It is not binding. All of space, or the entire length of a line, "exists" in the present. What it may have been in the past or may be in the future have no relevance. Translations, Reflections and Rotations are not limited by the impossibility of acquiring exact knowledge of the past, or comparing predictions with future events.

Space spreads itself before our eyes in its entirety. These considerations have nothing to do with the "inherent" attributes of space-time, but are fundamental to the way we think about them and the conceptual models we create.

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B. The other, even more important distinction between physical time and physical space, is how finite pieces of them are measured. One cannot simply state that "time is a measurable quantity". ***One has to build a machine to measure it.*** **The conventional name for a temporal measurement machine is a "clock".**

Aristotle cogently observes in the Physics that time is an attribute of motion. The measurement of time thus requires that motion be somehow quantified and a quantitative reading taken. The accumulation of "motion" implies that ***something*** **has to be**

moving; hence the necessity for a *machine* , some sort of dynamical system in isolation from external physical forces.

By the term "local universe" we will mean a region of space in which mechanics is essentially Newtonian. (Most of this discussion can be extended, with appropriate modifications, to an Einstein-Lorentz universe.) Of the two kinds of systems in a local universe which qualify as clocks, one of them, *uniform motion*, is a variant of the other, *periodic motion.*

To treat the space-time trajectory of a massive object in uniform motion as a "clock" is to commit an error of circular reasoning. The adjective "uniform" *means* that the object traverses equal segments of space in equal units of time. It doesn't tell us what *makes* temporal intervals "equal". Given the asymmetry of knowledge between initial and terminal measurements, one still needs a way of comparing the configuration of a machine at any point in its trajectory with the initial configuration. In addition to which one somehow has to be able to inspect an object in uniform motion at every place along its trajectory (there are Aleph-1 of them!) to verify that its velocity is unchanging. Ultimately one must posit a symmetry principle, invariance under spatial translation, then set up "fence posts" at regular intervals. These essentially convert uniform motion into the periodic motion of standard model for a clock. (To make the construction into a working machine, it must also be assumed, (as is implied by Galilean Relativity), that there is some way of sending signals "instantaneously", from each fence post back to the origin where the observer is situated.)

We will assume that, for a machine to be designated a clock means, both in theory and practice, that there exists a way of identifying a periodic return to an initial state in some autonomous sub-machine within its construction. Making such an identification requires a mixture of observation, theory and convention.

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The Metrization of Time

6 factors (A -F) must be present before one can affirm that a compact (closed and bounded) machine *H* can function as a clock.

A. *The initial state* : The state of every natural phenomena contributing to its motion is observable and can be *quantified* , that is to say, a set of numerical quantities unambiguously associated with that state description can be determined, at some instant τ , known as the *initial instant* and conventionally set to $\tau = 0$. These quantities can be gathered together into a set called the *initial state* or initial state description, S (0).

B. *Isolation*: *H may*, either in fact or in a thought experiment which does not violate laws of nature, be situated in a region of space that is unperturbed by any external force. In the equation presented in *The Measurement of Time*, (Audoin and Guinot, pg. 27) for the geocentric metric used in the ITS determination of the second, external tidal forces enter in the form of a "general" quantity U:

$$ds^2 = -c^2 d\tau^2$$
$$= -(1 - 2U/c^2)(dx^0)^2 + (1 + 2U/c^2)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2]$$

In this expression τ is proper time, $x0 = ct$, t being local geocentric time, and U is the sum of the Newtonian potential for the earth system of masses under consideration (taken as zero at infinity), plus a Newtonian tidal potential that vanishes at the center of gravity.

Although this gravitational potential enters as an external influence on geocentric time, it is local in the sense that it does not go beyond the solar system. Also, the quantity U is stable for practical purposes. One may therefore take the solar system as a whole as the system in isolation from general relativistic influences coming from the rest of the universe.

C. *Conservation*: For the interval of time in which *H* is active, one can ignore the effects of the 2nd Law of Thermodynamics. A functioning clock does not "run down". This has paradoxical implications that will be discussed further on.

D. *Periodicity*: S (0) is *exactly* reproduced at some later time which is otherwise unspecified, that is to say, *unpredictable in advance*: T. To say that there is a law of nature which can *predict* at what time T occurs, is to assume the existence of a metrizable time before the establishment of metrizability through this set of conditions.

E. *Causation* : Given the above conditions A, B, C, and D, the system *H* must return to the initial state S(0) infinitely often, in intervals of time T_1 , T_2 , T_3 ,...., T_n , ... , These are *defined* to be equal intervals of time, there being no way to compare them directly.

F. *Universality* : If $H1$ is such a machine, then any other machine H_2, satisfying conditions A, B, C, D and E, with the property that its state S_2 (0) measured at the same initial time 0, is exactly reproduced at simultaneously with S_1(0) of $H1$ at time T , will continue to pulse *simultaneously with H1* for as long as the two systems remain in isolation with respect to each other and to the rest of the universe. The dynamics of the two systems *H* and H_2 will also be extrapolated backwards in time under the assumption that their cyclings to their initial state *have coincided simultaneously* indefinitely into the past.

Condition F can be restated as follows:

If X and Y are two clocks with the same period, then the non-interacting union, $Z = [X \cup Y]$ is also a clock. The "union" of non-interacting systems can be unambiguously retrieved in the set union of their state descriptions $S_3 = S_1 \cup S_2$.

A space-time W in which systems obeying these 6 conditions exist, (pragmatically or theoretically) or can be constructed (the theoretical existence of constructible clocks) will be called *metrizable*. Universes which do not comply with all 6 conditions in which there is however, a demonstrable phenomenon called "motion", will be called *non-metrizable* .

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Commentary

In drawing up the list of numbers that go into the state description, one is led unavoidably to include velocities, that is to say, time derivatives. Two postulates and an axiom may provide a

way around the problem by restricting the entries in the state description set to configurations alone.

Postulate 1 Infinitesimal time intervals **: If two systems in isolation X and Y , are observed to** *configure identically* **at every instant in the closed time interval [t_1 , t_2] , $t_1 < t_2$, where t_2 is any time, arbitrarily small and distinct from t_1 , then they will configure identically for all future time, and will be assumed to have configured identically for all past time .**

Here the word "configuration", K (t) is the *frozen image* **of the system at the instant t. It therefore includes everything that doesn't involve derivatives, that is to say masses, positions, shapes, densities, charges. There is something of a problem, in that even the idea of a** *frozen photon* **may be meaningless. However, if one admits photons, one already admits a pair of functioning clocks, as will be discussed further below, and no axioms or postulates are needed to define metrizable or non-metrizable time.**

The advantages of this formulation are numerous. Not only does it avoid time derivatives, it enables one to move away from the "initial instant", which may be difficult to pin down, and speak about the initial duration, or initial infinitesimal duration.

Postulate 2 Invariance under time translation: **Let K be a dynamical system, with state description S at initial time t=0. Then it is allowed, in theory, that a machine K', with the same initial state description S can be constructed and launched at any specified time t 0 .**

This has very important consequences with regard to the "coupling" of clocks. What it *does* **allow is the coupling of a clock**

C_1 of period P_1 to a clock C_2 of period P_2 , by realizing the state description of the initial state of C_1 as a clock C_1' at the terminal instant of P_2 , then realizing the state description of the initial state of C_2 as a clock C_2' at the terminal instant of C_1', etc., to those "create" a coupled clock of period $P_1 + P_2 = P_3$.

What it *does not allow* , and this is the absolutely critical distinction between rulers and clocks, is the coupling of the *terminal instant* of P_1 with *the terminal instant* of P_2 , thereby creating a clock of period $P_4 = P_2 - P_1$. This follows directly from the irreversibility of temporal measurement. One would have to "guess" the point in the interval defined by P_2, at which to initiate C1 in order that a cycle of period P_1 should arrive simultaneously with a cycle of period P_2 at a mutual terminal point.

One might argue that the coupling of the *initial states* of C_1 and C_2 at the same time t = 0 can indirectly create a metric interval of duration $P_2 - P_1$, *between* the terminal observation of C_1 at P_1 and the terminal observation of C_2 of P_2 . There are two arguments against this:

(i) This is not a clock measurement. The system $C_1 + C_2$ at the terminal moment m_b of P2 has a state given by

$$S_3 = S_{C_1}(m_b) + S_{C_2}(m_b)$$

At the terminal moment m_a of C_1 it has the state

$$S_4 = S_{C_1}(m_a) + S_{C_2}(m_a)$$

,which is very different.

(ii) In order to use this construction to make a clock that measures the time interval $m_b - m_a$, one must somehow *push* m_a

back to the origin t = 0. This violates the irreversibility of time measurement. By contrast, rulers can be moved backwards and forwards without difficulty.

Time Asymmetry and Time Translation

In dealing with lengths the symmetry principle of invariance under linear translation can be used to show that the Euclidean line has no natural origin. However, (as Roger Penrose and others who study the light cone and the boundary of space-time will testify) , the time continuum *does* **have a natural origin, namely the present moment, which distinguishes two very different domains, past and future.**

This distinction is not only epistemological, but also in some sense topological. Thus it is possible to conceive of a sequence of instants in past time converging monotonically to a present moment as a limit point (lets call this a "Zeno sequence"); but a set of instants in future time converging to the present as a limit point goes against the customary orientation of the measurement of time, and cannot be imagined without strong theoretical assumptions.

In point of fact, unqualified time invariance is not a valid principle, given that the future is unknown and the past unknowable

Instead we propose replacing the principle of invariance under time translation with the following Axiom: (the notion of "identity" will be defined below):

Axiom I: **Identity is time invariant.**

An identity J is a set of numbers **which, in some Pythagorean or Platonic sense, stands outside of time. One lists all the observables entering into the mechanical description of a system K,**

as O_1, O_2, O_n, A collection of ordered pairs is thereby formed, in which the left-hand number refers to an observable in this list, and the right-hand number is a state, such as length, velocity, force, etc.

J is **a blueprint for reconstructing the mechanical system from which it derives. We assume that such complete descriptions are possible in theory, (though there may be some logical paradoxes involved in this assumption).**

Postulate 2 then states that this recipe can used to construct an identical and identically functioning machine at any specified time.

A *configuration identity Jc* is one that includes no time derivatives. We can restate Postulate 1 in terms of configuration identities and infinitesimal durations:

If systems X and Y in isolation from external forces, and from each other, have the same configuration identities IC (t), at every instant in some time interval, however small, they will be identical for as long as this isolation is maintained. **(Q**

The following quotation by Henri Poincaré is apt:

"When we use the pendulum to measure time, what postulate do we implicitly assume? It is that the duration of two identical phenomena is the same; or, if we prefer, that the same causes require the same time to produce the same effects."

(Audoin and Guinot, pg. 7, translating from Poincaré: *La valeur de la science* (Flammarion, 1906).)

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Commentary

It would appear that in many situations one cannot get away with eliminating observables that depend on time. Such necessary observables like velocities, forces. energies etc., are so much a part of what we consider the "identity" of a machine, that it does not seem possible to get rid of them.

However if it is possible to put the laws governing the functioning of a machine into the form of analytic functions, then the information provided by a configuration identity is sufficient to describe its behavior for all time.

This is because analytic functions have the property that the Taylor series evaluated at any point determines the configuration of a system throughout its entire domain.

Observe also that configuration identities are in agreement with the classical definition of a vector field. There is no time variable in the differential forms that define a vector field; time enters indirectly as a kind of hidden variable implicit in the equations.

In any discussion of the measurement of time in a universe W, the laws of governing motion in W enter in an indispensable way. One cannot, indeed, invoke the existence of a metrizable world without specifying the laws of motion (Galilean, Newtonian, Einsteinian, etc.), as well as the possibility (at least in theory), of constructing clocks in conformity to them.

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After one has verified that the 6 conditions (or their equivalents) of metrizability have been satisfied, one can begin to talk about the "topology", or "shape" of the time dimension. The

instrumentation, the system of clocks that will make the metric measurements has to come first. Since a machine has to be built for measuring time, one cannot talk about metrizability without introducing the laws of nature, that is to say, those of mechanics and fields.

Paradigmatic Example: *Cyclic Time*. **Let the universe W be such that every dynamical system is periodic, with a universal fixed period, T. Topologically one can speak of the "time continuum" as being circular. The unusual properties of cyclic time are treated in detail in Euclidean** *Time and Relativity* **. For a physical model, think of the overtime series above a ground pitch, produced by a vibrating string or column of air. The nodes of a string are the places which remain stationary when vibrating at a resonant frequency. The properties of cyclic time can then be analyzed in terms of the geometry of the rational points on a string of length 1.**

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Non-Metrizable Time

As stated above, a*non-metrizable* **time t, (or non-metrizable space-time W) is a universe in which one or more of the above six conditions cannot be fulfilled. Let us consider each case in turn:**

A* (Either *A doesn't hold in specific instances, or A holds nowhere, etc.* **) : In an A* universe there is no way to measure, perhaps even to define, an initial state for any or all of its systems at all moments. This description fits Quantum Theory, according to which position and momentum of a compact, connected particle cannot be measured simultaneously.**

In the spirit of George Gamov's "Mr. Tomkins in Wonderland", imagine a world with a very large Planck's constant, h. A mechanical clock has a single hand moving "clockwise". This motion is continuous; however if h is sufficiently large, it can move in discrete jumps without invalidating the model.

The hand is driven by an engine consuming an identical quantity of fuel with each jump. By measuring the amount of fuel remaining between jumps, one obtains a figure for the *angular momentum*. The "time" is ascertained by looking at the *position* of the hand. By the Uncertainty Principle, either this position is uncertain, making the reading inaccurate, or the momentum is uncertain, so that the criteria of "sameness of state", cannot be applied to the state of the hand.

Given a sufficiently large h, the time of the universe in which this clock is embedded, is non-metrizable.

Comment on the Time-Energy Relation

When measuring the length of an object, say from left to right, one can always , in theory in a Newtonian universe based on Euclidean geometry, return to the initial location and readjust the left end of the ruler in order to correct one's original at measurement. However there is no way to return to an initial moment to correct a temporal reading.

This implies an important modification of the energy-time version of the Uncertainty Principle: $\Delta E \Delta t > h$ **. This is normally interpreted to mean that the amount of energy required to make an accurate time measurement is inversely proportional to the degree**

of accuracy required. However, once a measurement has been made, there is no way one can return to repeat it, no matter how much energy one uses up. Here are some quotes from *The Quantum Challenge* (Greenstein and Zajonc, pg. 62):

"If a time Δt is required to measure the energy of a system, the result will be uncertain by an amount given by the energy-time uncertainty relation"

"...in certain situations, the energy-time relationship prevents us from finding out whether an effect comes after or before its cause."

In these situations the time continuum must be considered non-metrizable.

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B*: In a B* universe the constructibility of periodic systems either cannot be guaranteed or is ruled out. The space-time of Brownian motion is, once again, the paradigm.

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C*: In a C* universe it is forbidden to posit systems in isolation, even in theoretical thought experiments. We refer to this as a Leibniz-Kant universe. The following quotation is from my essay, "Algebraic Causation", (Part I, pg. 43):

"Gottfried *Leibniz and Immanuel Kant envisaged ..[that].. the entire cosmos, from inception to extinction, is entirely present at every point of space-time, in every instant and at every location. The mirroring of the Macrocosm in the Microcosm, the arbitrarily great in the vanishingly small, is universally present.* "

The universe models derived from solutions of the Einstein equations of General Relativity possess this property. Putting aside the action of the Hubble Expansion Field, a universal time cannot

be ascribed to the entire cosmos, since an infinite amount of time would be needed to "map" all of space-time, just to be able to state how it influences a single location, at which, presumably, one's clock is located.

The Hubble Expansion Field may admit some kind of regular "cosmic time variable". From Jim Peebles "Principles of Physical Cosmology, pg. 73:

".. the convenient coordinate labeling for the line element in the co-moving time-orthogonal construction.. In this construction one imagines a set of observers, each equipped with a clock synchronized relative to the neighboring observers, and each comoving with the mean motion of the material averaged over a neighborhood large enough to remove the local fluctuations away from homogeneity .. Homogeneity and isotropy require that the mean mass density and pressure are functions only of the world time t. "

This regularity has been challenged in recent decades with the discovery of an expansion attributable to dark energy.

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D*: In a D* universe there is no way to escape the 2nd Law of Thermodynamics. All clocks quickly run down. There is a paradox inherent in this characterization which extends to our own universe.

A clock can only "run down" *relative* **to another clock: how can one "know" if a clock is recycling "more" slowly or "more" quickly, unless there is a standard against which its pulsation can be compared? If C_1 and C_2 are both given the seal of approval as "reliable" "clocks" , and the hands of C_2 start moving "more slowly"**

than C_1 , one can either take C_1 as the "standard" giving the "forward direction of time", or one can take C_2 as "standard" and argue that the universe is speeding up !

By switching standard clocks, one has *reversed* the direction of time!

This paradox is solved in our universe by the Light Principle, which, by ascribing an unvarying fixed velocity of c to light, places the speed of the photon outside the 2nd Law. Photons are somehow constrained to "run down" only through energy loss, and in no other way. Since $E = Nh\nu$, where ν is the frequency and N the number of photons at that frequency, the 2nd Law finds expression either in the "weakening" of electro-magnetic frequency or in the reduction of the number of photons. Our universal clock "runs down" through the loss of convertible energy. Without something like the Light Principle, it is absurd to speak of the "universe running down". All motions would operate within the constraints of Galilean Relativity, which does not allow for a fixed standard clock.

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E*: *Unstable Causation* : An E* universe is "unstable" in the following way: there exist systems K , which execute a periodic return to initial conditions a certain number of times before spinning off in another configuration! One imagines some fuzziness in causation, manifesting itself over very long periods. Let us hypothesize that, in such a universe, one is unable to measure the difference between the periods of two clocks C1 and C_2, if their periods P1 and P_2 differ by $|P_1 - P_2| < \theta$. There exists

an integer N, however, such that the time interval = Nθ *is* discernible. ψ may be designated the *minimum discernible time quantum*. Suppose also that there exist systems such that, after M > N cycles one has a divergence of periods P_1^M, P_2^M given by

$$\left|P_1^M - P_2^M\right| > N\theta = \psi$$ **, This would be a model for an E* universe.**

The same question arises: which clock, C_1 of period P_1, or C2, with period P_2, is reliable? Perhaps they are both unreliable. Is there a third clock which can serve as the standard? Or is there in fact no standard reliable clock?

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F*: *Comparison*: In an F* universe, clocks from different regions of space can't be compared. Each clock is, however, authentic in its own region. An interesting example may be present in the functioning of the photon as a "double clock".

Louis DeBroglie and Erwin Schrödinger believed that all quantum phenomena could be described in terms of waves. Richard Feynman showed that, in Quantum Electrodynamics both electrons and photons could be treated as particles. Finally, Niels Bohr speaks of the particle picture and the wave picture as being "complementary images".

For our purposes it is sufficient to observe that a photon has a speed fixed by the speed of light c, and a frequency that relates to its energy by the relation E = Nhν . *Both* the *uniform velocity* of the photon, and the *unvarying frequency at a fixed energy* can function as clocks.

Using the constancy of speed of light to construct a clock X, one sets up a pair of mirrors at a fixed distance apart, and a beam of photons which travels from one to the other, bounces off and returns in a (theoretically) eternal periodic cycle.

Using the frequency of a photon as a clock machine, one calculates its energy, takes the inverse and multiplies by h. This gives the number of times the light wave cycles in a second. A machine Y that counts these is a clock. X and Y are very different instruments, linked by the constancy of the wavelength λ. In fact c = νλ = E/h, so hc/E = .

In an F* universe, these clocks might diverge. A variation in the wavelength of some standard color, say blue, might mean either

(1) There is no "red shift" of the spectrum, but the speed of light is increasing; or

(2) The speed of light isn't changing, but there is a "blue shift" affecting all the radiation of the universe.

Such worlds violate condition F: clocks which pulse synchronically for a long period will eventually diverge.

This picture finds its realization in the dual interpretation of the red shifts of distant galaxies as indicating either (1) The expansion of the universe, or (2) The tired light hypothesis. Once again, quoting from Peebles, pg. 225:

"In a tired light model, photons moving through apparently free space lose energy at the rate: $d\nu/dl = -H_0\nu$***, where*** H_0 ***is a constant, and dl is the proper displacement along the path of light in a static world."***

This paper should be considered a preliminary report only. At most it presents interesting ideas and indicates directions for further investigation.

Keep posted.

Roy Lisker, December 16, 2005

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