Irreversíble Motíon ín One Space Dímensíon An exercise in teleogeometry

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It has become standard practice in all the sciences to use simulations manufactured by computer graphics and the technologies of virtual reality. This, together with century-long history of narrative cinema have considerably modified our habits of thinking in images. The habit of "thinking in pictures", so disparaged by pure mathematicians but which is done by everyone, including them, in their actual research, has been extended in the modern world to the habit of "thinking in moving pictures"!

At the same time our minds find it easier to fix unchanging images of space-like geometric objects (triangles, trapezoids, intersections, convex figures, etc.) than it is to grasp a spatiotemporal process in its entirety. There are two reasons for this: one can, in theory and within limits in practice, travel in all directions in space, whereas experienced time (as opposed to theoretical time) , although it does not 'move', (no more than space moves: events move *through* time) can only be measured in the forward direction. Indeed one can define this forward direction as the way in which time is measured.

The second major obstacle in forming mental images in space-time, is that we are constrained to experience only one instant of time (at a time!), followed by an other instant, and so forth. Even the reasonable assumption that time is a continuum is thereby cast into doubt. In contrast to this, it is possible to see complete spatial objects in their wholeness, although it is of course more difficult to hold onto the picture of a full 3-dimensional shape, a piece of sculpture for example, than it is of a 2dimensional drawing. The invention of perspective in the 14th century greatly strengthened our capacities in this regard.

Not only is it difficult to grasp the temporal progress of a physical system, most of the devices we employ to do so entail translating a temporal picture into a spatial one. Useful if not

indispensable, these can on occasion lead to considerable inaccuracies . To take a simple example, the temporal trajectory of a bullet shot at an angle to the vertical in space can be portrayed as the Cartesian graph of a parabola. One might be tempted to confound this with the shape of its 2-dimensional trajectory in space, because the ratio of the horizontal motion in space with the forward direction in time is given by a velocity v that is assumed constant.

However, the trajectory in space is a "picture", while the trajectory in time (plotting, for example the vertical height against time) is a "movie". Although the *spatial* trajectory is indeed a parabola, a conic section, one cannot say that the *spatio-temporal* "shape" formed by the climb of a bullet against gravity is a conic section, for the simple reason that distance and time do not inter-relate through rotations based on the Pythagorean Theorem. The

"distance" $d = \sqrt{x^2 + t^2}$ has no natural equivalent as a quantity, in the same way that the expression $d(x,y) = \sqrt{x^2 + y^2}$ can be interpreted as the distance of a radius vector from the origin to a location on the spatial parabola, or the equation of a circle, or in the way that $d(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ is the distance to a point in

space. Spatial isualization in fact depends on these simple Euclidean relationships which have no meaning when applied to "shapes" combining time with space.

Several examples:

(1) Try to "picture" what a process governed by the equation $x^{2} + t^{2} = R^{2}$; $x = \sqrt{R^{2} - t^{2}}$ "looks like". Here we are not talking about the Cartesian plane image of a circle, but the "movie" created by watching a particle moving along the x-axis from time t = 0 to t=R. To watch this movie one must sense the speed at every instant, which means that one must consider the derivative:

$$\frac{dx}{dt} = \frac{-t}{\sqrt{R^2 - t^2}}$$

When t is 0, the velocity is 0. The particle moves back from the location R to the origin in time R. However, as one can see, *the velocity rises to minus infinity* ! What would normally be simple circular motion in a plane becomes *unpicturable* when one of the coordinates is time.

(2) Let U be a space-time of dimension 2+1: x, y for space, time for t.One can write down two "linear processes" which, where all the dimensions to be spatial, would be simple Euclidean planes:

(1)
$$Ax+By-t = C$$
;

(2) Dx + Ey - t = F

(A,B,C,D,E,F are all positive for simplicity)

The right way to visualize this system is as a motion picture showing a pair of lines moving around in the (x,y) plane. The slopes of these lines do not change as t increases to infinity; each line moves in parallel motion out to infinity as t goes to infinity. Finally, the "intersection" points as a function of time form a line given by (A-D)x+(B-E)=C-F. This is the correct way to visualize this system of equations

(3) Let's next take a look at the behavior of a "4 dimensional plane", with equation Ax+By +Cz-t = F. Being accustomed to perspective, we can make a picture in our minds of a stack of parallel planes (or rectangular portions of planes) rising away from the origin along a normal line:



Time

(4) Next, consider two 4- planes:

(*i*)
$$A_1x + B_1y + C_1z - t = F_1$$

(*ii*) $A_2x + B_2y + C_2z - t = F_2$

One can solve for x and y in terms of z and t. For *fixed t* one gets an intersection line which always falls on the same plane P, given by:

$$P: A_1x + B_1y + C_1z - F_1 = A_2x + B_2y + C_2z - F_2$$

Visualizing the way that this intersection line moves on the intersection plane as a function of time is a good mental exercise.

(4) Here is an example in the other direction. One can easily prove algebraically that two planes (2-dimensional) in 4-space can intersect in just a single point. Our spatial intuition finds it difficult to imagine this, but it's quite easy if one assumes that one of the dimensions is time.

To see this, note that, since a 3-plane in 4-space is formed by the intersection of two hyperplanes, each definable by a linear equation in 4 variables, a pair of planes can be formed by the simultaneous intersection of 4 equations in 4 variables:

$$3 - Plane U$$

(i) $A_1x + B_1y + C_1z - t = F_1$
(ii) $A_2x + B_2y + C_2z - t = F_2$
 $3 - Plane V$
(iii) $A_3x + B_3y + C_3z - t = F_3$
(iv) $A_2x + B_2y + C_2z - t = F_4$

The assumption that the coefficient of t is not zero has no effect on the generality of the description. One also assumes that the system of 4 linear equations is non-singular. When t is not at the unique solution, $t = \sigma$, of the system, the 4 equations, for a fixed t, represent 4 planes intersecting in 4 points and 6 lines, that is to say, a pyramid. At $t = \sigma$ the pyramid is crushed to a point which is the common intersection of the lines.

To visualize this as a process which can be filmed, one interprets the first pair of equations as the motion of a line moving across the plane given by the intersection of (i) and (ii), and likewise for equations (iii) and (iv). If the system is non- singular, the two lines must intersect in a single point in space at a single moment in time, σ .

The purpose behind presenting these examples is to suggest that some of the paradoxical properties of the physical dimension of time, notably those of measurements and reference frames, may be easier to understand through thought experiments involving the *spatial correlative* to time. Let's speculate that at a certain moment (*in time*!), there is a broken symmetry in one spatial direction, a line L = X'X with variable "x". such that all motion along L is irreversible. For convenience sake, we can assume that the motion of any material object along L must proceed in the 'positive' direction. This allows for the various possibilities which will be considered in this communication:

(A) The "universe" $U(x^+,t^+,v)$ in which this occurs is 2=1+1 dimensional, with space and time variables x and t, $v=dx/dt \ge 0$

(B) U is not Galilean; there is an "Absolute Reference Frame" ¹embedded in U, equivalent to the fixed stars. The Observer can plot his trajectory with respect to the fixed stars, as well as any other trajectory he is able to see. Nothing prevents the Observer from staying in one place as long as he wants, but if he begins to move it must be forward.

¹ the invariant Arf?

(C) One can consider models in which radiation is exempt from the conditions governing material motion.²

(D) Or light signals must also travel in this preferred direction. This means that no Observer can perceive anything to his right, i.e., in the positive direction. All signals can arrive only from the left.

(E) A y dimension along which motion is not constrained along one direction is added.

(F) Additional y and z dimensions , both of which are free from the irreversibility constraint are added.

(G) In the two-dimensional universe, $U(x^+,t^+,v)$, Galilean relativity is possible: that means that, from the vantage of every Observer in uniform motion, all other objects appear to be moving in the same direction.

A discussion of the properties of the above list of options, singly or in combination, ought to be sufficient for a preliminary paper.

² As an aside, the models we actually use for our own universe seem very strange: although light is constrained to move at a fixed speed, the universal expansion of material bodies, the galaxies for example, can accelerate them to any speed!

Symmetry breaking in a 2-D universe U (x,t)

In our first example, U is a Cartesian product of two dimensions, on both of which motion is irreversible. We also assume an absolute reference frame.

For convenience sake, the fixed stars are infinite in number, distributed along the x-axis in a regular fashion. This is not essential; what is important is that every Observer can find a fixed star somewhere to his left, relative to which he can plot a trajectory in space-time independent of his motion.

Radiation and matter are assumed to share the same restraint: they must move in the positive direction in both time and space. However, we do not at this point restrict signals to a limiting velocity c. Signals can travel at any finite speed. ³

Setting one of the fixed stars as origin, 0 = (0,0), consider 3 observers O_1 , O_2 , O_3 , at locations $0 < x_1 < x_2 < x_3$. Place ourselves for the moment at O_2 . Since radiation and matter can only move in the forward direction, O_2 can have no knowledge of O_3 . If, however O_2 is moving with a great velocity than O_3 , he will

³ One should never allow infinite speeds because they imply that something, whether radiation or light, can be in two places at the same time.

eventually overtake him, after which he can receive signals of any velocity from O_3 .

 O_2 can be aware of O_1 , if O_1 sends out signals with a velocity greater than the difference between the velocities of O_1 and O_2 . However, if O_1 is moving more slowly than O_2 , he will appear to be receding. Such relativity of motion can of course be corrected by reference to the fixed stars.

This behavior tells us something about time in ordinary space-time: we cannot know the past, it can only be reconstructed. However, if there is a signal of known velocity coming from some past event, at some distance d in space, we can know what happened at a past time, t, but only with respect to entities at the distance d. Thus, the totality of our "knowledge" at any time t=0, is a combination of what is happening instantaneously, and what has happened in the past, obtainable from signals moving more quickly than our own motion relative to the source of the signal.

The future cannot be known others than through hypothesis and predictions. To summarize:

 O_2 is ignorant of:

(a) Objects O_1 to the left, traveling at a speed less than O_2 , which send out signals which travel at a speed less than the relative motion of O_1 to O_2 .

(b) All objects O_3 to the right, whatever the relative speed.

 O_2 can have knowledge of :

Objects O_1 to the left which send out signals which travel at a speed greater than the relative motion of O_1 to O_2 . If there is a finite upper bound on the total speed of any object, radiation and particles (remember, this is a universe with an absolute frame), then someone traveling at that limiting speed can never know anything about the universe in which it is traveling, save through collision with objects (to the right) moving more slowly.

One can get a rough idea of how to experience such a universe, by imagine that one is inside a space ship where it is impossible to see anything of what is happening outside in front. A window at the back allows one to collect light-rays from stationary sources, or sources traveling in the same direction. The "universe" up front manifests itself only through the banging of massive bodies at the front of the space ship, which could equally well be interpreted as hammers, or rocks falling down from somewhere. Thus one could be excused for thinking that the universe does not extend beyond the front of the space ship.

This also corresponds to our perception of time. Future events just "show up", like the rocks hitting the front of the space ship. We invent laws and make predictions to anticipate the future, but this is not the same thing as "knowing the future", which implies a complete mastery of causation over all time, which is impossible.

The situation is not hopeless however: objects moving from left to right at an absolute speed faster than our own , will overtake us and move on. If we were able to derive their laws of motion from the signals sent to us while they are to the left, we could predict how they will interact with each other when they travel off to the right. Later when we once more come into contact with them , if we speed up or they slow down, we can verify our predictions against their actual behavior.

Collisions and Mechanics

In such a space there must be some inevitable modifications of classical mechanics. Any collision between two masses, M_1 and M_2 must be such that each continues to move to the right relative to

the fixed stars, or remain stationary. If M_2 is stationary, while M_1 is moving up to it from the left with velocity v_1 , then M_1 must have a mass equal to or greater than M_2 to avoid recoil.

One can easily work out the conditions on masses and velocities if both are moving to the right. However, if, in one's model, one wishes to be able to ascribe arbitrary velocities to massive objects ⁴, then one must allow that the masses of all objects are equal or decrease sequentially as one moves from left to right:

$$M_1 \ge M_2 \ge M_3 \ge \dots$$

On the other hand, if one wants to be free to arbitrarily assign masses, then one has little choice but to propose models in which there are no collisions: everything moves from left to right with sequentially equal or increasing velocities. Given that the model is not Galilean, one might propose a wholly novel form of mechanical interaction, wave packets for example, that just pass through each other.

Pair-Particle Creation

Envisaging a universe in which the "CPT Theorem" holds: non-reversibility means that parity is not conserved in mirror-

⁴ A model in physics is not really a model unless one can drop one's thought experiments into it.

imaging. Since time is also assumed to be irreversible, one can postulate that the changes in charge associated with anti-matter translate into a backward motion to the left.

Thus, in pair particle creation, such as the spontaneous creation of an electron and positron, the electron must move to the right, the positron to the left. Adding a new spatial dimension "y" to the model allows one to posit that the burst of energy accompanying the collision of a positive with an up-coming electron can move up or down along the y-axis.

In a similar vein, embedding $U(x^+,t^+,v)$ into a 4-D space with additional (space reversible) dimensions y and z, suggests models in which spin J_x components are always positive. Since

$$[J_y,J_z] = -\frac{i}{2\pi}J_x$$

this imposes a natural orientation, therefore a non-conservation of parity, in this universe model.

Measurement

As I show in my papers

"Euclidean Time and Relativity":

http://philsci-archive.pitt.edu/archive/00001290/

, "Non-Metrizable Time" :

www.fermentmagazine.org/essays/nmtime.pdf

, and "Clocks and Rulers":

www.fermentmagazine/essays/uic.html

, the mono-directional asymmetry of time leads to strong limitations on time measurement. One of these is the following: If there are two clocks C_1 , C_2 , pulsing in periods $P_1 < P_2$, unless P_1 is a divisor of P_2 , P_2/P_1 = Integer=n, then it is impossible to construct a clock with period P_2 - P_1 .

This is certainly not true for the measurement of spatial length. Given rulers of lengths $L_1 < L_2$, one can easily construct a ruler of length L_2 - L_1 , by aligning the left end points of L_1 and L_2 , and making a mark on L_2 at the place where it intersects the right end point of L_1 .

How does this work out in the universe $U(x^+,t^+,v)$? Allow that someone at rest can "pull forward" a ruler of length L, with endpoints A and B which can then be kept stationary. He can then travel to the location of endpoint B, and repeat the process. If he leaves a mark at each of the locations nL, of B, he sets up a scale of equally spaced units. He himself will never be able to use this scale, (even as one cannot go back in time) , but someone coming up from the left will be able to use it.

What happens if he has two rulers, of lengths $L_1 < L_2$? If he is permitted to carefully align their left end-points, A_1 , A_2 before laying them down, he can walk to the location of the end-point, B_1 , of L_1 . Here he uses a (metaphorical) knife, to "make a cut" in the longer ruler, thereby creating a ruler of length $L_2 - L_1$. In this respect, therefore, the situation is better than the one for clocks. This is because one is allowed to "pull up", the leftmost end of a ruler to where one is situated. One cannot "pull" the past tick of a clock up to the present. However, he cannot use his new ruler, of length $L_3 = L_2 - L_1$ to create a ruler of length kL_1 , where 0 <k<1 is a real number.

This is possible however, by means of the Euclidean Algorithm, whenever motions along X'X are reversible:

Theorem: Assume:

(1)Motion in any spatial direction is reversible.

(2) Given a ruler of length L, it is always possible to construct a ruler of length $L^* < L$.

Then one *can* approximate a ruler of length kL to any degree of accuracy.

Proof: Lay off L* along L as many times as possible, until one reaches an n such that $(n+1)L^* > L$. Then if there is a remainder L $nL^* = L^{**}$, one can lay this off along L. Iterate the process. If somewhere along the line L**..* turns out to be an exact divisor of L, one can use the fact that it is always possible to construct a smaller ruler to create one of length M <L**...* Say at stage n the reduced ruler makes F(n) divisions of L. Counting along L, one makes a mark m_n at a point G(n), such that G(n)/F(n) is the closest rational approximation to k.

The sequence of marks m_n will converge to k as n goes to infinity. Q.E.D.

This procedure cannot be applied if motion along the direction of measurement is irreversible. The application of the Euclidean Algorithm is based on the possibility of being able to move freely back and forth to various end-points along the way. Even at the first stage, in order to calculate the number n at which $(n+1)L^*>L$, one has to go beyond the rightmost endpoint of L. But one can't go back to it, to find out where it is!

Conclusion: In irreversible 1-space, although it is possible construct a length which is equal to the difference of two lengths,

one cannot use the Euclidean Algorithm to create a ruler which is in a given ratio to a given ruler.

In particular, there is no way to bisect the length of a ruler, unless one already has a ruler of length exactly equal to ¹/2ⁿ that of the given ruler.

What happens if the other two spatial dimensions y and z allow for reversible motions? Well, one has to allow that forward rotations from y (or z) are possible. From a ruler placed along the yaxis, one can construct a sub-ruler of the desired length, which can then be rotated in the forward direction onto the x-axis. If there are two reversible dimensions y and z, one can "use ruler and compass" to construct a length of the desired size, which can then be rotated, etc.

Another way of measuring distance in U, is to set up an equivalence between distance and matter. One does this by imagining that a car driven a certain distance will use up a certain amount of fuel. The distance from Middletown to New York can be measured in units of gallons of gasoline. By watching the contents of the fuel tank, one can lay off distances in terms of a matter equivalent. This assumes that all vehicles traveling through U are of the same make, never stall, get into accidents, etc. The matter/energy equivalence allows one to convert distances into units of energy. Since fuel is a quantity that can be augmented and depleted at will, this method can be used to create a "ruler" of any desired length, However, as before, the person making the measurements cannot directly avail himself of the benefits of his own work, which can only be used by persons to the length moving in his direction.

Summary: Measurement in $U(x^+,t^+,v)$

There are two ways of measuring distance in this model of a non-reversible, non-Galilean 2-dimensional universe:

(1) Pushing a ruler forward and recording the locations of its right endpoint as one comes to them. A construction which permits one to construct the difference of two lengths appears to be acceptable. Finding the mid-point of a given length, or any sublength in a given ratio, is shown to be impossible, at least by this method.

(2) Translating length into mass via the notion of the amount of combustible fuel used into getting from one place to another. The construction of a sub-length in a given ratio then depends on the possibility of measuring sub-units of fuel (coal, gasoline, etc.) All this is metaphorical, naturally; however translating "length" into a "matter equivalent" would be a way of measuring length.

(3) In a space of 3 dimensions U(x,y,t), in which motion along the y axis is not constrained, one can use the Euclidean Algorithm on a length in the y-direction to construct a sub-length in a given ratio, then "rotate" that sub-length forward into the x-direction

(4) In a space of 4 dimensions U(x,y,z,t), in which motions along both y and z axes are not constrained, one can make rulerand-compass constructions in the (y,z) plane to construct a desired sub-length, which can then be rotated onto the x-direction.

The Galilean Framework

There can be only one non-trivial (i.e. non-stationary) model for a Galilean Framework in which motions along the x-axis are constrained to go forward. I call it the Black Hole model, since, to all observers, the entire universe appears to be sucked into a Black Hole at the same moment in time!

In a Galilean model, motion, from the vantage of every observer , appears to be constrained to move from left to right. Each observer then imagines himself at rest in his own frame. Let observers be listed from left to right as ... $O_n, O_{n-1}, ..., O_1$ with "oneself" labeled as O_1 . Since everyone to one's left must appear to be moving forward, all the velocities are in the forward direction. However, if this is to be a Galilean frame, O_2 must also perceive all of the observers to *his* left as moving towards him. For O_3 to appear to be moving forward to both O_2 and O_1 , its velocity, as measured at O_1 , must be greater than the velocity of O_2 . Using induction one concludes that , from the vantage of any of these observers, the velocities must be perceived as monotonically increasing to the left.

No-one can "see" anything to his right. Observers just "disappear". Of course there is the static model in which nothing is moving. But if *even one* of the observers moves, they must all move!

This is because it is forbidden for any observer to overtake and pass any other observer. For example, if O_3 passes O_2 , O_2 will then appear to recede from O_3 , which is not allowed if *all* motions must be seen, by *all* observers, to be in the forward direction. Consequently, if the universe is not completely static, O_1 will experience the simultaneous collapse of everything to his left into his own location; and every other observer will experience the same thing!

It might be possible to "save the world" if there was some way of seeing what's happening to the right. But this simply will not work: the requirement that velocities increase as one moves to the left guarantees that observers to the right must eventually overtake and pass each other, which we have shown to be forbidden.

Summary: There are only 2 Galilean models:

(a) A totally static universe

(b) A collapsing universe. Each observer witnesses the

collapse of his left- universe at the same instant.

